Worked Examples

IN ACCORDANCE WITH

European Standards CEN/TC 250
Structural Eurocodes (EN 1990/EN 1991)

EUROCODES
SPREADSHEETS
Structural Design

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Beijing national stadium; Architect: Herzog & de Meuron, ArupSport, China Architectural Design & Research Group

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SOFTWARE EC2:

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Section 1 Eurocode 0 - EN1990

1.1 Foreword

EN 1990 "*Eurocode: Basis of structural design*" is the head document in the Eurocode suite. It describes the basis and general principles for the structural design and verification of buildings and civil engineering works including geotechnical aspects, the principles and requirements for safety and serviceability of structures and guidelines for related aspects of structural reliability in all circumstances in which a structure is required to give adequate performance, including fire and seismic events. Consisting of only one part, it is used with all the other Eurocodes (1 to 9) for design.

The Structural Eurocode programme comprises the following standards generally consisting of a number of Parts:

- EN 1990 Eurocode 0: Basis of Structural Design
- EN 1991 Eurocode 1: Actions on structures
- EN 1992 Eurocode 2: Design of concrete structures
- EN 1993 Eurocode 3: Design of steel structures
- EN 1994 Eurocode 4: Design of composite steel and concrete structures
- EN 1995 Eurocode 5: Design of timber structures
- EN 1996 Eurocode 6: Design of masonry structures
- EN 1997 Eurocode 7: Geotechnical design
- EN 1998 Eurocode 8: Design of structures for earthquake resistance
- EN 1999 Eurocode 9: Design of aluminium structures.

Eurocode standards recognise the responsibility of regulatory authorities in each Member State and have safeguarded their right to determine values related to regulatory safety matters at national level where these continue to vary from State to State.

1.2 National Standards implementing Eurocodes

The National Standards implementing Eurocodes will comprise the full text of the Eurocode (including any annexes), as published by CEN, which may be preceded by a National title page and National foreword, and may be followed by a National annex. The National annex may only contain information on those parameters which are left open in the Eurocode for national choice, known as Nationally Determined Parameters, to be used for the design of buildings and civil engineering works to be constructed in the country concerned, i.e.:

- values and/or classes where alternatives are given in the Eurocode
- values to be used where a symbol only is given in the Eurocode
- country specific data (geographical, climatic, etc.), e.g. snow map
- the procedure to be used where alternative procedures are given in the Eurocode.

It may also contain:

- decisions on the application of informative annexes
- references to non-contradictory complementary information to assist the user to apply the Eurocode.

1.3 National annex for EN 1990

This standard gives alternative procedures, values and recommendations for classes with notes indicating where national choices may have to be made. Hence the National Standard implementing EN 1990 should have a National annex containing all Nationally Determined Parameters to be used for the design of buildings and civil engineering.

1.4 Verification tests

EN1990.XLS. 4.4 MB. Created: 5 January 2013. Last/Rel.-date: 6 March 2013. Sheets:

— Splash

— Annex A1-B

— Annex C

— Annex D.

EXAMPLE 1-A- Target reliability index - **test 1**

Given: Target reliability index (1 year): $\beta_1 = 4,7$ (ultimate limit state: see tab. C2-EN1990). Find the probability of failure P_f (see ta. C1-EN1990) related to β and the value of β for a different reference period (say 100 years).

[*Reference sheet:* Annex C]-[*Cell-Range: A1:O1-A23:O23*].

Solution: In the Level II procedures (see Figure C1-EN1990 - *Overview of reliability methods*), an alternative measure of reliability is conventionally defined by the reliability index β which is related to the probably of failure P_f by:

$$P_f = \Phi(-\beta)$$

where Φ is the cumulative distribution function of the standardised Normal distribution.

The general formula for the probability density function of the Normal distribution is:

$$f(x) = \frac{\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]}{\sigma\sqrt{2\pi}} = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$$

where μ *is the "location parameter" and* σ *is the "scale parameter". The case where* $\mu = 0$ *and* $\sigma = 1$ *is called the Standard Normal distribution. The equation for the standard normal distribution is:*

$$f(x) = \frac{\exp\left[-\frac{x^2}{2}\right]}{\sqrt{2\pi}} = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}.$$

The probability of failure P_f *can be expressed through a performance function g such that a structure is considered to survive if g > 0 and to fail if g ≤ 0:* $P_f = \text{Prob}(g \le 0)$.

If g is Normally distributed, β *is taken as* $\beta = \mu_g/\sigma_g$ *(where* μ_g *is the mean value of g, and* σ_g *is the standard deviation), so that:* $\mu_g - \beta\sigma_g = 0$ *and* $P_f = \text{Prob}(g \le 0) = \text{Prob}(g \le \mu_g - \beta\sigma_g)$.

The cumulative distribution function (CDF) $\Phi(-\beta)$ *of a random variable is the probability of its value falling in the interval* $[-\infty; \beta]$, *as a function of x.*

The CDF of the standard normal distribution, usually denoted with the capital Greek letter Φ, *is the integral:*

$$\Phi(\beta) = \int_{-\infty}^{\beta} f(x)dx = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{4,7} e^{-\frac{x^2}{2}}dx.$$

END NOTE

For $\beta = 4,7 = \beta_1$ (1 year):

$$\Phi(4,7) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{4,7} e^{-\frac{x^2}{2}}dx \approx 10^{-6}.$$

For a reference period of n = 100 years the reliability index β_n is (see eq. C.3-EN1990):

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n \quad \rightarrow \Phi(\beta_n) = [\Phi(4,7)]^{100} \Rightarrow [10^{-6}]^{100} = 10^{-4} \Rightarrow \beta_n = 3,7,$$

where $\Phi^{-1}(\beta_n) = \beta_n$ is the inverse of the cumulative distribution function. The quantile of the standard normal distribution is the inverse of the cumulative distribution function.

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example-end

EXAMPLE 1-B- Approach for calibration of design values (section C7-EN1990) - **test 2**

Given: Calculate the design values of action effects E_d and resistances R_d. Assume a target reliability index equal to $\beta = 4,8$. The standard deviations of the action effect and resistance are, respectively: $\sigma_E = 5,0$, $\sigma_R = 5,0$.

[*Reference sheet:* Annex C]-[*Cell-Range: A27:O27-A70:O70*].

Solution: The design values of action effects E_d and resistances R_d should be defined such that the probability of having a more unfavourable value is as follows [see (C.6a), (C.6b) EN1990]:

$$P(E > E_d) = \Phi(+\alpha_E\beta)$$

$$P(R \le R_d) = \Phi(-\alpha_R\beta).$$

substituting σ_E and σ_R into eq. (C.7)-EN1990, we obtain: $0,16 < \sigma_E/\sigma_R < 7,6$. The values of FORM sensivity factors α_E and α_R may be taken as $-0,7$ and $0,8$, respectively. This gives:

$$P(E > E_d) = \Phi(+\alpha_E\beta) = \Phi(-0,7\beta) = \Phi(-0,7 \times 4,8) = \Phi(-3,36) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-3,36} e^{-\frac{x^2}{2}} dx$$

$$P(R \le R_d) = \Phi(-\alpha_R\beta) = \Phi(-0,8\beta) = \Phi(-0,8 \times 4,8) = \Phi(-3,84) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-3,84} e^{-\frac{x^2}{2}} dx.$$

Using the given numerical data, we find (leading variable only):

$$P(E > E_d) = \Phi(+\alpha_E\beta) = 3,90 \times 10^{-4}$$

$$P(R \le R_d) = \Phi(-\alpha_R\beta) = 6,15 \times 10^{-5}.$$

When the action model contains several basic variables, for the accompanying actions the design value is defined by:

$$P(E > E_d) = \Phi(+\alpha_E 0,4\beta),$$

from which we obtain:

$$P(E > E_d) = \Phi(+\alpha_E 0,4\beta) = \Phi(-0,28\beta) = \Phi(-0,28 \times 4,8) = \Phi(-1,344) = 8,95 \times 10^{-2}.$$

example-end

EXAMPLE 1-C- Approach for calibration of design values (section C7-EN1990) - **test 3**

Given: Consider the same assumptions in the example above ($\beta = 4,8$). Assume $\sigma_E = 1,0$, $\sigma_R = 7,0$. Find the design values of action effects E_d and resistances R_d.

[*Reference sheet:* Annex C]-[*Cell-Range: A27:O27-A70:O70*].

Solution: The condition $0,16 < \sigma_E / \sigma_R < 7,6$ is not satisfied: $\alpha = \pm 1,0$ should be used for the variable with the larger standard deviation, and $\alpha = \pm 0,4$ for the variable with the smaller standard deviation.

The value of α is negative for unfavourable actions and action effects, and positive for resistances. Using these values of α, the design equations become:

$$P(E > E_d) = \Phi(+\alpha_E \beta) = \Phi(-0,4\beta) = \Phi(-0,4 \times 4,8) = 2,74 \times 10^{-2}$$

$$P(R \le R_d) = \Phi(-\alpha_R \beta) = \Phi(-1,0\beta) = \Phi(-4,8) = 7,93 \times 10^{-7}.$$

For the accompanying actions the design value is (smaller standard deviation):

$$P(E > E_d) = \Phi(+\alpha_E 0,4\beta) = \Phi(-0,4 \cdot 0,4\beta) = \Phi(-0,16 \times 4,8) = 2,21 \times 10^{-1}.$$

example-end

EXAMPLE 1-D- Approach for calibration of design values (section C7-EN1990) - **test 4a**

Given: Derive the design values of variables with a probability equal to 10^{-4} (reliability index around $\beta = 3,8$) using a Gumbel distribution. Assume:

$\mu_E = 30$; $\sigma_E = 1,0$

$\mu_R = 30$; $\sigma_R = 7,0$

(mean value and standard deviation of the action effect and resistance, respectively).

[*Reference sheet:* Annex C]-[*Cell-Range: A74:O74-A140:O140*].

Solution: Considering the same assumptions in the example above (condition $0,16 < \sigma_E / \sigma_R < 7,6$ not satisfied), it is seen that:

α_E	$-\alpha_E\beta$	σ_E/μ_E	$\Phi(-\alpha_E\beta)$	α_R	$-\alpha_R\beta$	σ_R/μ_R	$\Phi(-\alpha_R\beta)$
- 0,40	1,52	0,033	9,36 x 10^{-1}	1,00	- 3,80	0,233	7,23 x 10^{-5}

Table 1.1 Input data. See previous examples.

From table C3-EN1990 - *Design values for various distribution functions*, by using the Gumbel distribution with the given numerical data, it follows that:

$$a = \frac{\pi}{\sigma_R \sqrt{6}} = \frac{\pi}{7\sqrt{6}} = 0,183;\ u = \mu_R - \frac{0,577}{a} = 30 - \frac{0,577}{0,183} = 26,85$$

$$a = \frac{\pi}{\sigma_E \sqrt{6}} = \frac{\pi}{1\sqrt{6}} = 1,283;\ u = \mu_E - \frac{0,577}{a} = 30 - \frac{0,577}{1,283} = 29,55.$$

Therefore, it is (leading variable action):

$$X_{di,R} = u - \frac{1}{a} \cdot \ln\{-\ln[\Phi(-\alpha_R \beta)]\} = u - \frac{1}{0,183} \cdot \ln\{-\ln[7,235 \times 10^{-5}]\} = u - \frac{1}{0,183} \cdot 2,255$$

$$X_{di,\,R} = 26,85 - \frac{1}{0,183} \cdot 2,255 = 14,5$$

$$X_{di,\,E} = u - \frac{1}{a} \cdot \ln\{-\ln[\Phi(-\alpha_E\beta)]\} = u - \frac{1}{1,283} \cdot \ln\{-\ln[9,357\times10^{-1}]\} = u + \frac{1}{1,283} \cdot 2,712$$

$$X_{di,\,R} = 29,55 + \frac{1}{1,283} \cdot 2,712 = 31,7 .$$

example-end

EXAMPLE 1-E- Approach for calibration of design values (section C7-EN1990) - **test 4b**

Given: Considering the same assumptions in the example above (see tab. 1.1), derive the design values of action effects E_d with a probability equal to 10^{-4} using a Normal and a Log-normal distribution.

[*Reference sheet:* Annex C]-[*Cell-Range: A74:O74-A140:O140*].

Solution: Remembering that the condition $0,16 < \sigma_E/\sigma_R < 7,6$ is not satisfied, the design value of action effects for Normal distribution (see tab. C3-EN1990 - *Design values for various distribution functions*) becomes:

$$X_{di,\,E} = \mu - \alpha\beta\sigma = 30 + (0,40 \cdot 3,8 \cdot 1) = 31,5 .$$

Having calculated $V = \sigma_E/\mu_E = 1/30 = 0,033 < 0,2$, the design value of action effects for Log-normal distribution becomes:

$$X_{di,\,E} = \mu \cdot \exp(-\alpha\beta V) = 30 \cdot \exp(0,40 \cdot 3,8 \cdot 0,033) = 30 \cdot 1,051 = 31,6 .$$

example-end

EXAMPLE 1-F- Ψ_0 factors (section C10-EN1990) - **test 5**

Given: Use the expressions in tab. C4-EN1990 for obtaining the Ψ_0 factors in the case of two variable actions. Consider the following assumptions:

– reference period T = 50 years

– greater of the basic periods (for actions to be combined) $T_1 = 7$ years

– reliability index $\beta = 3,8$

– coefficient of variation V = 0,30 of the accompanying action (for the reference period).

[*Reference sheet:* Annex C]-[*Cell-Range: A144:O144-A189:O189*].

Solution: The distribution functions in Table C4 refer to the maxima within the reference period T. These distribution functions are total functions which consider the probability that an action value is zero during certain periods. The theory is based on the calculation of the inverse gamma distribution's probability density function of the extreme value of the accompanying action in the reference period.

The gamma distribution, like the Log-normal distribution, is a two-parameter family of continuous probability distributions. The general formula for the probability density function of the gamma distribution is:

$$f(x) = \frac{\left(\frac{x-\mu}{\beta}\right)^{\gamma-1}\exp\left(-\frac{x-\mu}{\beta}\right)}{\beta\Gamma(\gamma)}\,;\ x \ge \mu\,;\ \gamma, \beta > 0\,.$$

where γ *is the shape parameter,* μ *is the location parameter,* β *is the scale parameter, and* $\Gamma(\gamma)$ *is the "gamma function which" has the formula:*

$$\Gamma(\gamma) = \int_0^\infty t^{\gamma-1}e^{-t}dt.$$

The case where $\mu = 0$ *and* $\beta = 1$ *is called the "standard gamma distribution". The equation for the standard gamma distribution reduces to:*

$$f(x) = \frac{x^{\gamma-1}e^{-x}}{\Gamma(\gamma)}\,;\ x \ge 0\,;\ \gamma > 0\,.$$

The gamma distribution can be parameterized in terms of a shape parameter α *and an inverse scale parameter* $1/\gamma = \beta$*, called a rate parameter:*

$$g(x;\,\alpha,\beta) = \beta^\alpha\frac{1}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x},$$

With this parameterization, a $\mathrm{gamma}(\alpha, \beta)$ *distribution has mean* $\alpha\beta$ *and variance* $\alpha\beta^2$*. As in the log-normal distribution, x and the parameters* α *and* β *must be positive. The cumulative distribution function is the regularized gamma function:*

$$F(x) = P\{X \le x\} = \int_0^x \beta^\alpha\frac{1}{\Gamma(\alpha)}t^{\alpha-1}e^{-\beta t}\,dt.$$

The inverse gamma distribution's probability density function is defined over the support x > 0:

$$[g(x;\,\alpha,\beta)]^{-1} = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}e^{-\beta/x}.$$

Therefore, the inverse $F^{-1}(x)$ *of the cumulative distribution function* $F(x)$ *is the quantile of the standard gamma distribution:* $F^{-1}(x) = x$.

END NOTE

Ratio approximated to the nearest integer: $N_1 = T/T_1 = 50/7 = 7{,}14 \rightarrow 7$.

Shape parameter α (gamma distribution):

$$k = (\mu/\sigma)^2 = (1/V)^2 = V^{-2} = (0{,}30)^{-2} = 11{,}1 = \alpha$$

Scale parameter β (gamma distribution):

$$\lambda = \frac{\mu}{\sigma^2} = \frac{\mu}{V^2\mu^2} = V^{-2} = (0,30)^{-2} = 11,1 \Rightarrow \beta = \frac{1}{\lambda} = (11,1)^{-1}$$

From table C4-EN1990:

$$\beta' = -\Phi^{-1}\left\{\frac{\Phi(-0,7\beta)}{N_1}\right\} = -\Phi^{-1}\left\{\frac{\Phi(-0,7\cdot 3,8)}{N_1}\right\} = -\Phi^{-1}\left\{\frac{\Phi(-2,66)}{7}\right\} = 3,3$$

$$\Phi(0,4\beta') = \Phi(0,4\cdot 3,3) = \Phi(1,32) = 0,9066\,;\ [\Phi(0,4\beta')]^{N_1} = 0,9066^7 = 0,5034$$

$$\Phi(0,7\beta) = \Phi(0,7\cdot 3,8) = \Phi(2,66) = 0,9961\,;\ [\Phi(0,7\beta)]^{N_1} = 0,9961^7 = 0,9730\,;$$

$$-\ln(\Phi(0,7\beta)) = -\ln(0,9961) = 0,0039\,,$$

$$\Phi(-0,4\beta') = \Phi(-0,4\cdot 3,3) = \Phi(-1,32) = 0,0934$$

$$-N_1\Phi(-0,4\beta') = -7\cdot 0,0934 = -0,6538$$

$$\exp[-N_1\Phi(-0,4\beta')] = \exp[-0,6538] = 0,5200$$

$$\Phi(0,28\beta) = \Phi(0,28\cdot 3,8) = \Phi(1,06) = 0,8563\,;\ -\ln(\Phi(0,28\beta)) = -\ln(0,8563) = 0,1551$$

Quantiles (for $\alpha = 11,1$, $\beta = 1/11,1$):

$$F_S^{-1}\{[\Phi(0,4\beta')]^{N_1}\} = F_S^{-1}\{0,5034\} = 0,9727\,,\ F_S^{-1}\{[\Phi(0,7\beta)]^{N_1}\} = F_S^{-1}\{0,9730\} = 1,6546$$

$$F_S^{-1}\{\Phi(0,7\beta)\} = F_S^{-1}\{0,9961\} = 1,9797$$

$$F_S^{-1}\{\exp[-N_1\Phi(-0,4\beta')]\} = F_S^{-1}\{0,5200\} = 0,9850\,.$$

Substituting the numerical data listed above into expressions in table C4-EN1990, we find:

a) General expression:

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{F_S^{-1}\{[\Phi(0,4\beta')]^{N_1}\}}{F_S^{-1}\{[\Phi(0,7\beta)]^{N_1}\}} = \frac{0,9727}{1,6546} = 0,588\,.$$

b) Approximation for very large N_1:

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{F_S^{-1}\{\exp[-N_1\Phi(-0,4\beta')]\}}{F_S^{-1}\{\Phi(0,7\beta)\}} = \frac{0,9850}{1,9797} = 0,497\,.$$

c) Normal (approximation):

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{1+(0,28\beta-0,7\ln N_1)V}{1+0,7\beta V} = \frac{1+(0,28\cdot 3,8-0,7\ln 7)(0,30)}{1+0,7\cdot(3,8\cdot 0,30)} = \frac{0,9106}{1,798} =$$

$$= 0,506\,.$$

d) Gumbel (approximation):

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{1-0,78V\{0,58+\ln[-\ln(\Phi(0,28\beta))]+\ln N_1\}}{1-0,78V\{0,58+\ln[-\ln(\Phi(0,7\beta))]\}} =$$

$$= \frac{1-0,78(0,30)\{0,58+\ln[0,1551]+\ln 7\}}{1-0,78(0,30)\{0,58+\ln[0,0039]\}} = \frac{1-0,78(0,30)\{0,58-1,8637+1,9459\}}{1-0,78(0,30)\{0,58-5,5468\}} =$$

$$= \frac{0,8450}{2,1622} = 0,391\,.$$

example-end

EXAMPLE 1-G- D7.2 Assessment via the characteristic value - **test 6**

Given: Find the design value of the property X considering already known the ratio η_d/γ_m between the design factor of the conversion factor and the partial factor of the material.

Suppose a simple random sample of size n = 30 is drawn from a population having mean μ and standard deviation σ (see table below). Suppose the original distribution is normal.

n	x_i	n	x_i
1	19,3	16	17,3
2	19,8	17	19,2
3	20,1	18	22,4
4	20,4	19	16,0
5	20,3	20	15,0
6	19,3	21	15,6
7	18,0	22	18,2
8	17,4	23	17,4
9	21,3	24	19,2
10	19,4	25	16,3
11	20,2	26	15,3
12	20,5	27	14,0
13	21,0	28	13,0
14	22,3	29	15,3
15	18,5	30	16,5

Table 1.2 Sample results (n = 30). Reference Sheet: Annex D. Cell-Range B50:B64 - E50:E64.

Find the mean, variance, standard deviation and the coefficient of variation of the sampling distribution. Rounding to the first decimal.

[*Reference sheet:* Annex D]-[*Cell-Range: A1:O1-A82:O82*].

Solution: Mean of the n = 30 sample results:

$$m_X = \frac{(19,3 + 19,8 + 20,1 + 20,4 + \ldots + 13,0 + 15,3 + 16,5)}{30} = 18,3 .$$

Variance:

$$s_x^2 = \frac{1}{30-1}[(19,3 - 18,3)^2 + (19,8 - 18,3)^2 + (20,1 - 18,3)^2 + \ldots + (16,5 - 18,3)^2] = 6,0 .$$

Standard deviation: $s_x = \sqrt{6,0} = 2,45$.

Coefficient of variation:

$$V_X = \frac{s_x}{m_x} = \frac{2,45}{18,3} = 0,13 .$$

Values of k_n for the 5% characteristic value for n = 30 (see tab. D1-RN1990):

$$k_n = \begin{cases} 1,67 & V_x \text{ known} \\ 1,73 & V_x \text{ unknown} \end{cases}$$

Design value of the property X:

$$X_d = \eta_d \frac{X_{k(n)}}{\gamma_m} = \frac{\eta_d}{\gamma_m} m_x(1 - k_n V_x) = \frac{\eta_d}{\gamma_m} \cdot 18,3 \cdot \left(1 - \begin{bmatrix} 1,67 \\ 1,73 \end{bmatrix} 0,13\right) = \frac{\eta_d}{\gamma_m} \cdot \begin{cases} 14,3 & V_x \text{ known} \\ 14,2 & V_x \text{ unknown} \end{cases}$$

having considered already known the ratio η_d / γ_m.

example-end

EXAMPLE 1-H- D7.2 Assessment via the characteristic value - **test 7**

Given: Considering the same sample result in the example above (see tab. 1.2) and supposing the original distribution is Log-normal, find the design value of a property X considering already known the ratio η_d / γ_m. Rounding to the first decimal.

[*Reference sheet:* Annex D]-[*Cell-Range: A84:O84-A125:O125*].

Solution: Estimated value m_y for $E(\Delta)$:

$$m_y = \overline{\Delta} = \frac{1}{n}\sum_{i=1}^{n} \ln(\delta_i) = \frac{1}{n}\sum_{i=1}^{n} \Delta_i = \frac{1}{30}[\ln(19,3) + \ln(19,8) + \ldots + \ln(15,3) + \ln(16,5)] = 2,897$$

Estimated value s_Δ for σ_Δ:

$s_y = s_\Delta = \sqrt{\ln(V_\delta^2 + 1)} \approx V_\delta = 0,09$ [*input:* (If V_δ is known from prior knowledge)].

Estimated value s_Δ for σ_Δ [(If V_δ is unknown from prior knowledge)]:

$$s_y = s_\Delta = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - m_y)^2} =$$

$$= \sqrt{\frac{1}{29}[(2,960 - 2,897)^2 + (2,986 - 2,897)^2 + \ldots + (2,602 - 2,897)^2]} = 0,139 .$$

Values of k_n for the 5% characteristic value for n = 30 (see tab. D1-EN1990):

$$k_n = \begin{cases} 1,67 & V_x \text{ known} \\ 1,73 & V_x \text{ unknown} \end{cases}$$

Solution: For the standard evaluation procedure the following assumptions are made:
– the resistance function is a function of a number of independent variables X
– a sufficient number of test results is available
– all relevant geometrical and material properties are measured
– there is no statistical correlation between the variables in the resistance function
– all variables follow either a Normal or a log-normal distribution.

Step 1. Develop a design model, say in general:

$$r_{ti} = A_i \cdot B_i \cdot C_I \cdot D_I \cdot H_I \cdot L_I \cdot M_I \cdot N_I \cdot Q_I \cdot T_I .$$

Step 2. Compare experimental and theoretical values.
The points representing pairs of corresponding values $(r_{ti}; r_{ei})$ are plotted on a diagram (see data on table 1.3):

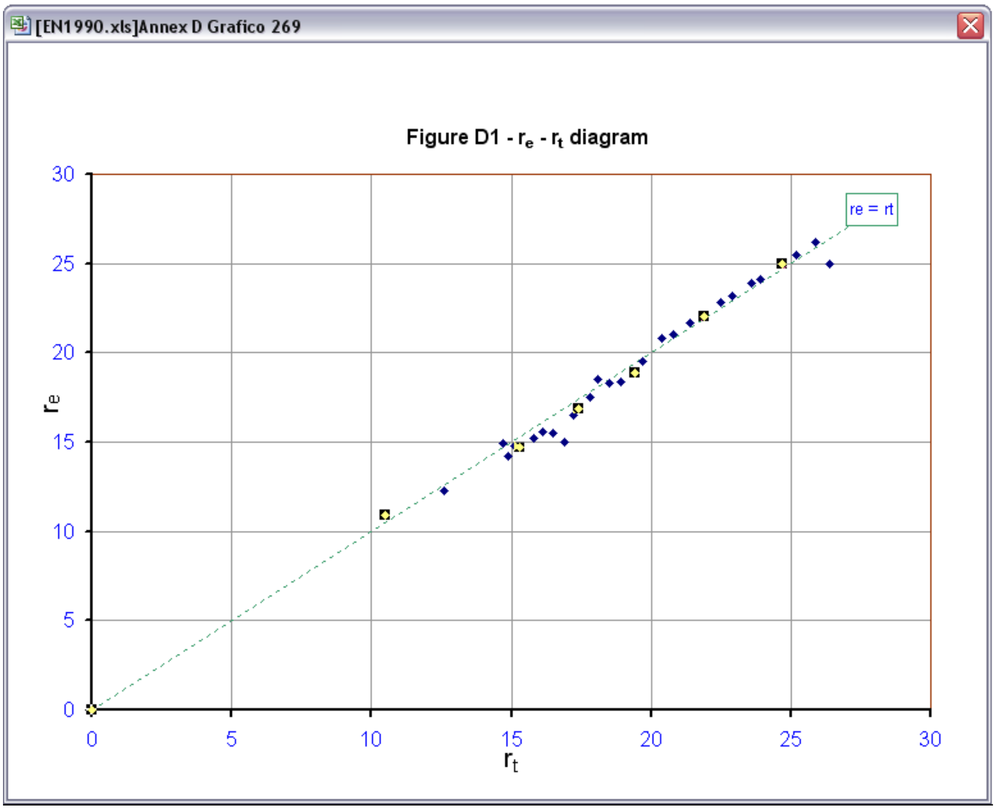


Figure 1.1 Windows screen image: figure D1-EN1990 (r_e - r_t diagram).

As we can see in figure 1.1, all of the points lie on the line $\theta = \pi/4$ (equation $r_e = r_t$). It means that the resistance function is reasonably exact and complete: a sufficient correlation is achieved between the theoretical values and the test data.

Step 3. Estimate the mean value correction factor b.

$$\sum_{i=1}^{n} r_{ei}r_{ti} = 10,5 \times 10,9 + 12,6 \times 12,3 + \ldots + 26,4 \times 25,0 = 11401$$

$$\sum_{i=1}^{n} r_{ti}^2 = (10,5)^2 + (12,6)^2 + (14,7)^2 + \ldots + (25,9)^2 + (26,4)^2 = 11501$$

$$b = \frac{\sum_{i=1}^{n} r_{ei}r_{ti}}{\sum_{i=1}^{n} r_{ti}^2} = \frac{11401}{11501} = 0,991.$$

Probabilistic model of the resistance $r = br_t\delta$. The mean value of the theoretical resistance function, calculated using the mean values $\underline{X}_m$ of the basic variables, can be obtained from:

$$r_m = br_t(\underline{X}_m)\delta = bg_{rt}(\underline{X}_m)\delta.$$

Step 4. Estimate the coefficient of variation of the errors.

The error term δ_i for each experimental value r_{ei} should be determined from expression (D9-EN1990):

$$\delta_i = \frac{r_{ei}}{br_{ti}}.$$

From which, using the given numerical data into table 1.3, we find (rounding to three decimal places):

$$\delta_1 = \frac{r_{e1}}{br_{t1}} = \frac{10,9}{0,991 \times 10,5} = 1,047;\ \Delta_1 = \ln\delta_1 = \ln(1,047) = 0,046;$$

$$\delta_2 = \frac{r_{e2}}{br_{t2}} = \frac{12,3}{0,991 \times 12,6} = 0,985;\ \Delta_2 = \ln\delta_2 = \ln(0,985) = -0,015;$$

$$\delta_3 = \frac{r_{e3}}{br_{t3}} = \frac{14,9}{0,991 \times 14,7} = 1,023;\ \Delta_3 = \ln\delta_3 = \ln(1,023) = 0,023;$$

...

$$\delta_{30} = \frac{r_{e30}}{br_{t30}} = \frac{25,0}{0,991 \times 26,4} = 0,956;\ \Delta_{30} = \ln\delta_{30} = \ln(0,956) = -0,045.$$

Substituting the above numerical data into expressions (D.11), (D.12), (D13), we find:

$$\bar{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\Delta_i = \frac{1}{n}\sum_{i=1}^{n}\ln(\delta_i) = \frac{(0,046 - 0,015 + 0,023 + \ldots - 0,045)}{30} = -0,005$$

$$s_\Delta^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - \bar{\Delta})^2 = \frac{(0,046 + 0,005)^2 + (-0,015 + 0,005)^2 + \ldots + (-0,045 + 0,005)^2}{29} = 0,001$$

Coefficient of variation V_δ of the δ_i error terms:

$V_\delta = \sqrt{\exp(s_\Delta^2) - 1} = \sqrt{\exp(0,001^2) - 1} = 0,032$.

Step 5. Analyse compatibility.

The compatibility of the test population with the assumptions made in the resistance function should be analysed. If the scatter of the $(r_{ei}; r_{ti})$ values is too high to give economical design resistance functions, this scatter may be reduced. To determine which parameters have most influence on the scatter, the test results may be split into subsets with respect to these parameters. When determining the fractile factors k_n (see step 7), the k_n value for the sub-sets may be determined on the basis of the total number of the tests in the original series.

Step 6. Determine the coefficients of variation V_{Xi} of the basic variables.

Consider, for example, the design model for the theoretical resistance r_{ti} as represented by the following relation (bearing resistance for bolts):

$r_{ti} = 2,5 d_i t_i f_{ui} \equiv 2,5\ B_i \cdot C_i \cdot D_i; (A_i = A = 2,5 = \text{cost})$.

The resistance function above covers all relevant basic variables $\underline{X}$ that affect the resistance at the relevant limit state. The coefficients of variation V_{Xi} will normally need to be determined on the basis of some prior knowledge. Therefore, let us say:

1) coefficient of variation $V_d = 0,04$ of the basic variable of the bolt's diameter;

2) coefficient of variation $V_t = 0,05$ of the b. v. of the thickness of the connected part;

3) coefficient of variation $V_{fu} = 0,07$ of the b. v. of the ultimate tensile strength of the materials.

Step 7. Determine the characteristic value r_t of the resistance.

The resistance function for j (= 4) basic variables is a product function of the form:

$r = br_t\delta = b\{A \cdot B \cdot C \cdot D\}\delta$.

Coefficient of variation V_r:

$$V_r^2 = (V_\delta^2 + 1)\left[\prod_{i=1}^{j}(V_{Xi}^2 + 1)\right] - 1 = (V_\delta^2 + 1) \cdot (V_a^2 + 1) \cdot (V_d^2 + 1) \cdot (V_t^2 + 1) \cdot (V_{fu}^2 + 1) - 1$$

having considered $V_A = 0$ for the constant $A = 2,5$. Therefore, rounded to two decimal places, we find:

$$V_r^2 = (0,032^2 + 1) \cdot (0^2 + 1) \cdot (0,04^2 + 1) \cdot (0,05^2 + 1) \cdot (0,07^2 + 1) - 1 = 0,01$$

$$V_\delta = \sqrt{\exp(s_\Delta^2) - 1} = 0,032$$

$$V_{rt}^2 = \sum_{i=1}^{n} V_{xi}^2 = V_a^2 + V_d^2 + V_t^2 + V_{fu}^2 = 0 + 0,04^2 + 0,05^2 + 0,07^2 = 0,009.$$

The number of test is limited (n = 30 < 100). In this case the characteristic resistance r_k should be obtained from [see equation (D.17)-EN1990]:

$r_k = bg_{rt}(\underline{X}_m)\exp(-k_\infty\alpha_{rt}Q_{rt} - k_n\alpha_\delta Q_\delta - 0,5Q^2)$ with:

$$Q = \sqrt{\ln(V_r^2+1)} = \sqrt{\ln(0,01+1)} = 0,100$$

$$Q_{rt} = \sqrt{\ln(V_{rt}^2+1)} = \sqrt{\ln(0,009+1)} = 0,095\,;\ \alpha_{rt} = Q_{rt}/Q = (0,095)/(0,100) = 0,95$$

$$Q_\delta = \sqrt{\ln(V_\delta^2+1)} = \sqrt{\ln(0,032^2+1)} = 0,032\,;\ \alpha_\delta = Q_\delta/Q = (0,032)/(0,100) = 0,32\,.$$

Values of k_n for the 5% characteristic value for n = 30 (see tab. D1-EN1990):

$$\begin{bmatrix} k_\infty \\ k_n \end{bmatrix} = \begin{cases} 1,64 & \text{for } n \to \infty \\ 1,73 & V_x \text{ unknown} \end{cases}$$

Substituting the numerical data into expressions above, we find the characteristic value of the resistance:

$$r_k = r_m \exp(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\delta Q_\delta - 0,5Q^2) =$$

$$= r_m \exp[-(1,64\cdot 0,95\cdot 0,095)-(1,73\cdot 0,32\cdot 0,032)-0,5\cdot 0,100^2] = r_m\cdot\exp(-0,171) =$$

$$= r_m\cdot\exp(-0,171) = r_m\cdot 0,84$$

Here the characteristic value r_k is represented as being proportional to its mean r_m.

example-end

EXAMPLE 1-J-D8.3 Standard evaluation procedure (Method (b)) - **test 9**

Given: Considering the same assumptions in the example above, determine the design value of the resistance by taking account of the deviations of all the variables.

[*Reference sheet:* Annex D]-[*Cell-Range: A387:O387-A413:O413*].

Solution: In this case the procedure is the same as in D8.2, excepted that step 7 is adapted by replacing the characteristic fractile factor k_n by the design fractile factor $k_{d,n}$ equal to the product $\alpha_R\beta$ assessed at $0,8\times 3,8 = 3,04$ as commonly accepted (see Annex C-EN1990) to obtain the design value r_d of the resistance.

For the case of a limited number of tests (herein n = 30 < 100) the design value r_d should be obtained from:

$$r_d = b g_{rt}(\underline{X}_m)\exp(-k_{d,\infty}\alpha_{rt}Q_{rt} - k_{d,n}\alpha_\delta Q_\delta - 0,5Q^2)$$

where:

$k_{d,n}$ is the design fractile factor from table D2 for the case "V_X unknown"

$k_{d,\infty}$ is the value of $k_{d,n}$ for $n\to\infty$ $[k_{d,\infty} = 3,04]$.

The value of $k_{d,n}$ for the ULS design value (leading) is 3,44 (see table D2-EN1990).

Therefore, we get:

$$r_d = r_m \exp(-k_{d,\infty}\alpha_{rt}Q_{rt} - k_{d,n}\alpha_\delta Q_\delta - 0,5Q^2) =$$

$$= r_m \exp[-(3,04\cdot 0,95\cdot 0,095)-(3,44\cdot 0,32\cdot 0,032)-0,5\cdot 0,100^2] = r_m\cdot\exp(-0,315) =$$

$$= r_m\cdot\exp(-0,315) = r_m\cdot 0,73$$

having represented r_d as being proportional to its mean.

Dividing the characteristic value by the design value we obtain:

$$\gamma_R = \frac{r_k}{r_d} = \frac{r_m \cdot 0,84}{r_m \cdot 0,73} \approx 1,15$$

having estimated V_δ from the test sample under consideration (see data in tab. 1.3).

example-end

EXAMPLE 1-K-D8.4 Use additional prior knowledge - **test 10**

Given: Determine the characteristic value r_k of resistance when:

– only one further test is carried out.

– two or three further tests are carried out.

Suppose that the maximum coefficient of variation observed in previous tests is equal to $V_r = 0,09$.

[*Reference sheet:* Annex D]-[*Cell-Range: A419:O419-A438:O438*].

Solution: If only one further test is carried out, the characteristic value r_k may be determined from the result r_e of this test by applying (D.24-EN1990):

$$r_k = r_e \cdot \eta_k = r_e \cdot 0,9\exp(-2,31V_r - 0,5V_r^2) = r_e \cdot 0,9\exp(-2,31 \cdot 0,09 - 0,5 \cdot 0,09^2) = r_e \cdot 0,73$$

where η_k is a reduction factor applicable in the case of prior knowledge.

If two or three further tests are carried out, the characteristic value r_k may be determined from the mean value r_{em} of the test results by applying (D.26-EN1990):

$$r_k = r_e \cdot \eta_k = r_e \cdot \exp(-2,0V_r - 0,5V_r^2) = r_e \cdot \exp(-2,0 \cdot 0,09 - 0,5 \cdot 0,09^2) = r_e \cdot 0,83$$

provided that each extreme (maximum or minimum) value r_{ee} satisfies the condition: $|r_{ee} - r_{em}| \le 0,10 r_{em}$.

example-end

1.5 References [Section 1]

BS EN 1990 - *Eurocode 0: Basis of structural design*, 1 July 2002

European Committee for Standardization (2001) *Eurocode: Basis of Structural Design*, CEN, Brussels, EN 1990

Ferry-Borges, J. and Casteneta, M. (1972) *Structural Safety*. Laboratorio Nacional de Engenheria Civil, Lisbon.

Section 2 Eurocode 1 - EN1991-1-1

2.1 Foreword

The Eurocode standards provide common structural design rules for everyday use for the design of whole structures and component products of both a traditional and an innovative nature. Unusual forms of construction or design conditions are not specifically covered and additional expert consideration will be required by the designer in such cases.

The National Standards implementing Eurocodes will comprise the full text of the Eurocode (including any annexes), as published by CEN, which may be preceded by a National title page and National foreword, and may be followed by a National annex.

EN 1991-1-1 gives design guidance and actions for the structural design of buildings and civil engineering works, including the following aspects:

— densities of construction materials and stored materials

— self-weight of construction elements, and

— imposed loads for buildings.

EN 1991-1-1 is intended for clients, designers, contractors and public authorities. EN 1991-1-1 is intended to be used with EN 1990, the other Parts of EN 1991 and EN 1992 to EN 1999 for the design of structures.

2.2 National annex for EN 1991-1-1

This standard gives alternative procedures, values and recommendations for classes with notes indicating where National choices have to be made, therefore the National Standard implementing EN 1991-1-1 should have a National Annex containing all Nationally Determined Parameters to be used for the design of buildings and civil engineering works to be constructed in the relevant country.

2.3 Distinction between Principles and Application Rules

Depending on the character of the individual clauses, distinction is made in this Part between Principles and Application Rules. The Principles comprise:

DYNAMIC ACTIONS. Actions which cause significant acceleration of the structure or structural members shall be classified as dynamic actions and shall be considered using a dynamic analysis.

2.5 Representation of actions

Characteristic values of densities of construction and stored materials should be specified. Mean values should be used as "*characteristic values*". Annex A gives mean values for densities and angles of repose for stored materials. The self-weight of the construction works should in most cases, be represented by a single characteristic value and be calculated on the basis of the nominal dimensions and the characteristic values of the densities. When a range is given it is assumed that the mean value will be highly dependent on the source of the material and may be selected considering each individual project.

The determination of the characteristic values of self-weight, and of the dimensions and densities shall be in accordance with EN 1990, 4.1.2.

For the determination of the imposed loads, floor and roof areas in buildings should be sub-divided into categories according to their use (see Tables 6.1-6.2 EN 1991-1-1). The imposed loads specified in this part are modelled by uniformly distributed loads, line loads or concentrated loads or combinations of these loads.

The categories of loaded areas, as specified in Table 6.1, shall be designed by using characteristic values q_k (uniformly distributed load) and Q_k (concentrated load). Values for q_k and Q_k are given in Table 6.2. Where a range is given in this table, the value may be set by the National annex. The recommended values, intended for separate application, are underlined. q_k is intended for determination of general effects and Q_k for local effects. The National annex may define different conditions of use of this Table.

For the design of a floor structure within one storey or a roof, the imposed load shall be taken into account as a free action applied at the most unfavourable part of the influence area of the action effects considered. Where the loads on other storeys are relevant, they may be assumed to be distributed uniformly (fixed actions). To ensure a minimum local resistance of the floor structure a separate verification shall be performed with a concentrated load that, unless stated otherwise, shall not be combined with the uniformly distributed loads or other variable actions. Imposed loads from a single category may be reduced according to the areas supported by the appropriate member, by a reduction factor α_A according to 6.3.1.2(10). In design situations when imposed loads act simultaneously with other variable actions (e.g actions induced by wind, snow, cranes or machinery), the total imposed loads considered in the load case shall be considered as a single action. When the imposed load is considered as an accompanying action, in accordance with EN 1990, only one of the two factors ψ (EN 1990, Table A1.1) and α_n (6.3.1.2 (11)) shall be applied. The imposed loads to be considered for serviceability limit state verifications should be specified in accordance with the service conditions and the requirements concerning the performance of the structure. For structures susceptible to vibrations, dynamic models of imposed loads should be considered where relevant. The design procedure is given in EN 1990 clause 5.1.3.

2.6 Rapresentative values

For each variable action there are four representative values. The principal representative value is the *characteristic value* and this can be determined statistically or, where there is insufficient data, a nominal value may be used. The other representative values are combination, *frequent* and *quasi-permanent*; these are obtained by applying to the characteristic value the factors ψ_0, ψ_1 and ψ_2 respectively. A semi-probabilistic method is used to derive the ψ_j factors, which vary depending on the type of imposed load. Further information on derivation of the ψ_j factors can be found in Appendix C of the Eurocode.

2.7 Ultimate limit state

The ultimate limit states are divided into the following categories:

— **EQU** Loss of equilibrium of the structure.

— **STR** Internal failure or excessive deformation of the structure

— or structural member.

— **GEO** Failure due to excessive deformation of the ground.

— **FAT** Fatigue failure of the structure or structural members.

The Eurocode gives different combinations for each of these ultimate limit states. For the purpose of this publication only the STR ultimate limit state will be considered.

2.8 Verification tests

EN1991-1-1.XLS. 6.5 MB. Created: 5 January 2013. Last/Rel.-date: 20 March 2013. Sheets:

— Splash

— CodeSec6

— Annex A

— Annex B.

EXAMPLE 2-L- Reduction factors for imposed loads - **test 1**

Given: A five-storey building is dedicated only or primarily for use as offices. Each deck below the roof is constituted by a reinforced concrete floor slab simply supported on beams, columns and walls, and has to carry an imposed load (*characteristic value*) of $q_k = 5,0\ \mathrm{kN/m^2}$ (Category of use: C3). Suppose that the mean influence area supported by a single beam is approximately $A = 75\ \mathrm{m^2}$. Determine both the reduction factors: α_A for beams (**case a**: see eq. 6.1-EN1991-1-1) and α_n (**case b**: see eq. 6.2-EN1991-1-1) for columns and walls (say, of the first floor).

[*Reference sheet:* CodeSec6]-[*Cell-Range: A86:O86-A115:O115*].

Solution: From table 6.1 - Categories of use:

"***C3***: *Areas without obstacles for moving people, e.g. areas in museums, exhibition rooms, etc. and access areas in public and administration buildings, hotels, hospitals, railway station forecourts.*"

From table 6.2 - Imposed loads on floors, balconies and stairs in buildings:

Category C3: $3,0 \le q_k[\mathrm{kN/m^2}] \le 5,0$; $4,0 \le Q_k[\mathrm{kN}] \le 7,0$.

Case a. Imposed loads from a single category may be reduced according to the areas supported by the appropriate member (e.g. a beam), by a reduction factor α_A according to 6.3.1.2(10). Therefore, the reduction factor α_A is applied to the q_k values for imposed loads C3 for floors: $\alpha_A q_k \le q_k$.

Factor according to EN 1990 (see Annex A1, Table A1.1): $\psi_0 = 0,7$.

Assuming $A = 75\ \mathrm{m^2}$ the influence area of the beam, with $A_0 = 10,0\ \mathrm{m^2}$ (see NOTE 1-6.3.1.2) eq. 6.1 becomes:

$$\alpha_A = \frac{5}{7}\psi_0 + \frac{A_0}{A} = \frac{5}{7} \cdot 0,7 + \frac{(10\ \mathrm{m^2})}{(75\ \mathrm{m^2})} = 0,63 \le 1,0 .$$

with the restriction for categories C and D: $\alpha_A \ge 0,6$.

Case b. Where imposed loads from several storeys act on columns and/or walls, the total imposed loads may be reduced by a factor α_n according to 6.3.1.2(11) and 3.3.1(2)P. The area is classified according to table 6.1 into category C. Therefore, in accordance with 6.2.2(2), for columns and/or walls the total imposed loads from n = 4 storey (same category C3: $q_k = 5,0\ \mathrm{kN/m^2}$) may be multiplied by the reduction factor:

$$\alpha_n = \frac{2 + (n-2)\psi_0}{n} = \frac{2 + (4-2) \cdot 0,7}{4} = 0,85 ,$$

where $n = 4$ is the number of storeys (> 2) above the loaded structural elements (in this case, columns and walls of the first floor) from the category C3. In other words:

$$N_{k,tot} = (q_{k,roof} + n\alpha_n q_k) \cdot A_{i,col} = [q_{k,roof} + 4 \cdot (0,85 q_k)] \cdot A_{i,col}$$

where $A_{i,col}$ is the influence area of the single column/wall of the first floor.

example-end

EXAMPLE 2-M- Imposed loads on floors, balconies and stairs in buildings - **test 2a**

Given: A series of 500 mm deep x 250 mm wide reinforced concrete beams spaced at 4,00 m centres and spanning 6,50 m support a 200 mm thick reinforced concrete slab. If the imposed floor loading is $3,0\ \mathrm{kN/m^2}$ and the load induced by the weight of concrete is

$$V_{Ed} = \frac{1}{2} q_d L = \frac{1}{2}(49,22)(6,50) = 159,97 \text{ kN}.$$

PreCalculus. (see Figure 2.2)

Case a) Characteristic loads (dead + imposed):

UDL: $q_k = (3,125)/(4,00) + (0,20 \cdot 25) + 3,00 = 8,78 \text{ kN/m}^2$.

Beam's length: $L = 6,50$ m. Width floor supported: $i = 4,00$ m. Partial safety factors for all load (dead and imposed) set equal to 1,45 (approx.).

example-end

EXAMPLE 2-N- Imposed loads on floors, balconies and stairs in buildings - **test 2b**

Given: A simply supported steel beam spans L = 7 m and supports an ultimate central point load of $Q_d = 170$ kN from secondary beams. In addition it carries an ultimate UDL of $q_d = 1,13$ kN/m resulting from its self-weight. Find ultimate bending moment and shear.

[*Reference sheet:* CodeSec6]-[*Cell-Range: A47:O47-A84:O84*].

Solution: The maximum ultimate moment and shear are given by, respectively:

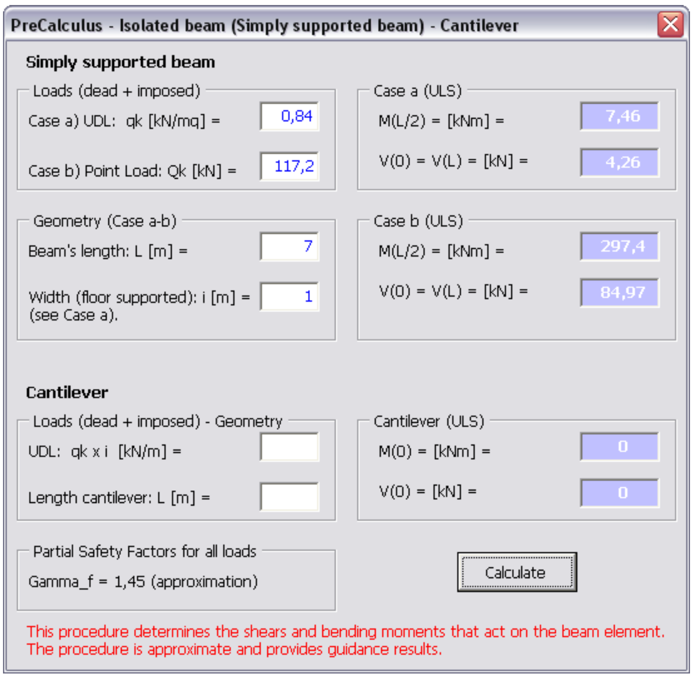


Figure 2.3 PreCalculus Excel form: procedure for a quick pre-calculation.

$$M_{Ed} = \frac{Q_d L}{4} + \frac{1}{8} q_d L^2 = \frac{(170)(7)}{4} + \frac{1}{8}(1,13)(7)^2 = 297,5 + 6,92 = 304,42 \text{ kNm}.$$

$$V_{Ed} = \frac{Q_d}{2} + \frac{1}{2} q_d L = \frac{170}{2} + \frac{1}{2}(1,13)(7) = 85 + 3,96 = 88,96 \text{ kN}.$$

PreCalculus (see Figure 2.3).

Case a) Characteristic loads (dead + imposed):

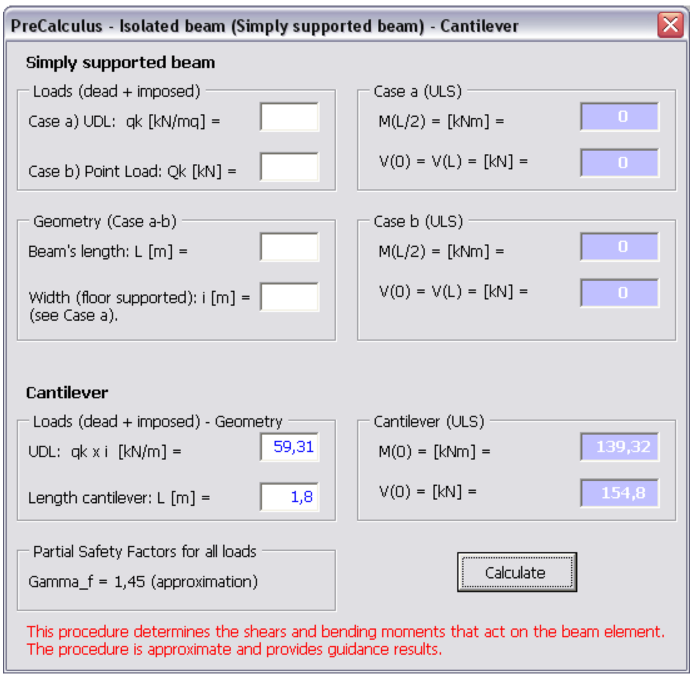
UDL: $q_k = \frac{1,13}{1,35} = 0,84 \text{ kN/m} = (0,84 \text{ kN/m}^2) \cdot (1 \text{ m})$ (self-weight). Where the width floor supported (i [m]) must be set equal to 1 (see Figure 2.3).

Point Load: $Q_k = 170/(1,45) = 117,2 \text{ kN}$ (imposed loads approximation).We find (see form above): $M_{Ed} = 297,4 + 7,46 = 304,86 \text{ kNm}$, $V_{Ed} = 84,97 + 4,26 = 89,23 \text{ kN}$.

example-end

EXAMPLE 2-O-Imposed loads on floors, balconies and stairs in buildings - **test 2c**

Given: A cantilever steel beam, length L = 1,80 m, supports a total UDL including its self-weight of $q_d \cdot i = 86 \text{ kN}$ (design value). Suppose the lateral torsional buckling resistance moment of the **I** beam is equal to $M_{buckl} = 100 \text{ kNm}$. Check if the beam section is adequate.


PreCalculus - Isolated beam (Simply supported beam) - Cantilever

Simply supported beam

Loads (dead + imposed)
Case a) UDL: qk [kN/mq] =
Case b) Point Load: Qk [kN] =

Case a (ULS)
M(L/2) = [kNm] = 0
V(0) = V(L) = [kN] = 0

Geometry (Case a-b)
Beam's length: L [m] =
Width (floor supported): i [m] = (see Case a).

Case b (ULS)
M(L/2) = [kNm] = 0
V(0) = V(L) = [kN] = 0

Cantilever

Loads (dead + imposed) - Geometry
UDL: qk x i [kN/m] = 59,31
Length cantilever: L [m] = 1,8

Cantilever (ULS)
M(0) = [kNm] = 139,32
V(0) = [kN] = 154,8

Partial Safety Factors for all loads
Gamma_f = 1,45 (approximation)

Calculate

This procedure determines the shears and bending moments that act on the beam element.
The procedure is approximate and provides guidance results.

Figure 2.4 PreCalculus Excel form: procedure for a quick pre-calculation.

[*Reference sheet:* CodeSec6]-[*Cell-Range: A150:O150-A199:O199*].

Solution: The maximum ultimate moment and shear are given by, respectively:

$$M_{Ed} = \frac{1}{2}(q_d \cdot i)L^2 = \frac{1}{2}(86)(1,80)^2 = 139,32\ \text{kNm}\ ;\ V_{Ed} = q_d L = (86)(1,80) = 154,80\ \text{kN}\,.$$

The beam section is not adequate: $M_{Ed} = 139,32\ \text{kNm} > (M_{buckl} = 100\ \text{kNm})$.

PreCalculus (see Figure 2.4).

Cantilever) Characteristic loads (dead + imposed):

$q_k \cdot i = (86\ \text{kN})/(1,45) = 59,31\ \text{kN}$ (approximation).

Length cantilever: $L = 1,80\ \text{m}$.

Obviously, we find (see form above): $M_{Ed} = 139,32\ \text{kNm}$, $V_{Ed} = 154,80\ \text{kN}$.

example-end

EXAMPLE 2-P- Areas for storage and industrial activities - Actions induced by forklifts - **test 3**

Given: A 250 mm thick reinforced concrete floor slab is simply supported on beams and columns. Concrete beams, which are 500 mm deep by 250 mm wide, spanning L = 4,50 m and spaced at 3,50 m centres have to carry an imposed load at least of $Q_k = 40\ \text{kN}$ (axle characteristic load) due to forklifts and transport vehicles on pneumatic tyres (class of forklifts: FL2, see table 6.6-EN1991-1-1). Considering all the imposed loads to be placed at the more unfavourable location, quickly assess the beam's stresses due to bending moment and shear.

[*Reference sheet:* CodeSec6]-[*Cell-Range: A150:O150-A199:O199*].

Solution: From table 6.5 - EN1991-1-1:

Class of Forklift	Net weight [kN]	Hoisting load [kN]	Width of axle a [m]	Overall width b [m]	Overall length l [m]
FL2	31	15	0,95	1,10	3,00

Table 2.4 Dimension of forklift according to class FL2 (from table 6.5-EN 1991-1-1).

Dynamic magnification factor $\varphi = 1,40$ (pneumatic tyres). Dynamic characteristic value of the action: $Q_{k,\,dyn} = \varphi Q_k = 1,40 \cdot (40\ \text{kN}) = 56\ \text{kN}$.

Horizontal loads due to acceleration or deceleration of forklifts may be taken as 30% of the vertical axle loads Q_k (dynamic factors need not be applied):

$H_{k,\,dyn} = 0,30 \cdot Q_k = 0,30 \cdot (40\ \text{kN}) = 12\ \text{kN}$.

From table A.1 - *Construction materials-concrete and mortar* (Annex A - EN1991-1-1):

concrete (normal weight and normal percentage of reinforcing):

Partial safety factors for all load (dead and imposed) set equal to 1,45 (approx.).

example-end

EXAMPLE 2-Q- Vehicle barriers and parapets for car parks - **test 4**

Given: Find the horizontal force F (in kN), normal to and uniformly distributed over any length of 1,50 m of a barrier for a car park, required to withstand the impact of a vehicle.

[*Reference sheet:* Annex B]-[*Cell-Range: A1:O1-A70:O70*].

Solution: **Clause B(3)-EN1991-1-1-Annex B.**

Suppose:

– deformation of the vehicle: $\delta_c = 100 \text{ mm} = 100 \cdot 10^{-3} \text{ m}$

– deformation of the barrier: $\delta_b = 0 \text{ mm}$ (rigid barrier)

– velocity of the vehicle normal to the barrier: $v = 4,5 \text{ m/s}$

– gross mass of the vehicles using the car park: $m \le 2500 \text{ kg}$. (The mass of $m = 1500$ kg is taken as being more representative of the vehicle population than the extreme value of 2500 kg).

The horizontal characteristic force, normal to and uniformly distributed over any length of 1,50 m of a barrier for a car park, is given by:

$$F_k = \frac{\frac{1}{2}mv^2}{(\delta_c + \delta_b)} = 0,5\frac{(1500) \cdot (4,5)^2}{100 \cdot 10^{-3}} = 151875 \text{ N} = 151,88 \text{ kN} > 150 \text{ kN} \text{ (rigid barrier).}$$

From table A1.2(B) - Design values of actions (STR/GEO) (Set B) - EN1990: $\gamma_{Q,1} = 1,5$ (leading variable action). Hence, the horizontal design force is given by:

$F_d = \gamma_{Q,1}F_k = 1,5 \cdot (151,88) = 227,82 \text{ kN}$.

Bumper eight above finish floor level (FFL): $h_d = 375 \text{ mm}$ (design height).

Bending moment (design value): $M_{Ed} = F_d \cdot h_d = (227,82) \cdot (0,375) = 85,43 \text{ kNm}$.

Clause B(4)-EN1991-1-1-Annex B.

The car park has been designed for vehicles whose gross mass exceeds 2500 kg.

Actual mass of the vehicle for which the car park is designed: say $m = 3000 \text{ kg}$. Therefore, we get:

$$F_k = \frac{\frac{1}{2}mv^2}{(\delta_c + \delta_b)} = 0,5\frac{(3000) \cdot (4,5)^2}{100 \cdot 10^{-3}} = 303750 \text{ N} = 303,75 \text{ kN} \text{ (rigid barrier).}$$

Design value: $F_d = \gamma_{Q,1}F_k = 1,5 \cdot (303,75) = 455,63 \text{ kN}$.

Bumper eight above finish floor level (FFL): say $h_{ac} = 550 \text{ mm}$ (actual height).

Bending moment (design value): $M_{Ed} = F_d \cdot h_{ac} = (455,63) \cdot (0,550) = 250,60 \text{ kNm}$.

Clause B(6)-EN1991-1-1-Annex B.

Barriers to access ramps of car parks have to withstand one half of the force determined in B(3) or B(4) acting at a height of 610 mm above the ramp:

Design value (ref. clause B(3)):

$$F_d = \gamma_{Q,1} \cdot (0,5 \cdot F_k) = 1,5 \cdot (0,5 \cdot 151,88) = 113,91 \text{ kN}.$$

Bending moment (design value):

$$M_{Ed} = F_d \cdot h_d = (113,91) \cdot (0,610) = 69,49 \text{ kNm}.$$

Design value (ref. clause B(4)):

$$F_d = \gamma_{Q,1} \cdot (0,5 \cdot F_k) = 1,5 \cdot (0,5 \cdot 303,75) = 227,81 \text{ kN}.$$

Bending moment (design value):

$$M_{Ed} = F_d \cdot h_d = (227,81) \cdot (0,610) = 138,96 \text{ kNm}.$$

Clause B(7)-EN1991-1-1-Annex B.

Opposite the ends of straight ramps intended for downward travel which exceed 20 m in length the barrier has to withstand twice the force determined in B (3) acting at a height of 610 mm above the ramp. Therefore, we get:

design value (ref. clause B(3)):

$$F_d = \gamma_{Q,1} \cdot (2,0 \cdot F_k) = 1,5 \cdot (2,0 \cdot 151,88) = 455,64 \text{ kN}.$$

Bending moment (design value):

$$M_{Ed} = F_d \cdot h_d = (455,64) \cdot (0,610) = 277,94 \text{ kNm}.$$

example-end

2.9 References [Section 2]

BS EN 1991-1-1 Eurocode 1: Actions on structures – Part 1-1: General actions – Densities, self-weight and imposed loads - 29 July 2002. (Incorporating corrigenda December 2004 and March 2009).

EN 1991-1-1:2002 - Eurocode 1: Actions on structures - Part 1-1: General actions - Densities, self-weight, imposed loads for buildings - CEN/TC 250 - Structural Eurocodes.

International Organization for Standardization (1999) Bases for Design of Structures - Notations - General Symbols. ISO, Geneva, ISO 3898.

The Concrete Centre. *How to Design Concrete Structures using Eurocode 2*, 2006.

Trevor Draycott, *Structural Elements Design Manual*, Butterworth Heinemann, 1999.

Section 3 **Eurocode 1**
EN 1991-1-2

3.1 General

The methods given in this Part 1-2 of EN 1991 are applicable to buildings, with a fire load related to the building and its occupancy. This Part 1-2 of EN 1991 deals with thermal and mechanical actions on structures exposed to fire. It is intended to be used in conjunction with the fire design Parts of prEN 1992 to prEN 1996 and prEN 1999 which give rules for designing structures for fire resistance. This Part 1-2 of EN 1991 contains thermal actions related to nominal and physically based thermal actions. More data and models for physically based thermal actions are given in annexes.

In addition to the general assumptions of EN 1990 the following assumptions apply:

- any active and passive fire protection systems taken into account in the design will be adequately maintained
- the choice of the relevant design fire scenario is made by appropriate qualified and experienced personnel, or is given by the relevant national regulation.

The rules given in EN 1990:2002, 1.4 apply. For the purposes of this European Standard, the terms and definitions given in EN 1990:2002, 1.5 and the following apply.

3.2 Terms relating to thermal actions

FIRE COMPARTMENT. Space within a building, extending over one or several floors, which is enclosed by separating elements such that fire spread beyond the compartment is prevented during the relevant fire exposure.

FIRE RESISTANCE. Ability of a structure, a part of a structure or a member to fulfil its required functions (load bearing function and/or fire separating function) for a specified load level, for a specified fire exposure and for a specified period of time.

EQUIVALENT TIME OF FIRE EXPOSURE. time of exposure to the standard temperature-time curve supposed to have the same heating effect as a real fire in the compartment.

EXTERNAL MEMBER. Structural member located outside the building that may be exposed to fire through openings in the building enclosure.

GLOBAL STRUCTURAL ANALYSIS (FOR FIRE). Structural analysis of the entire structure, when either the entire structure, or only a part of it, are exposed to fire. Indirect fire actions are considered throughout the structure.

MEMBER. Basic part of a structure (such as beam, column, but also assembly such as stud wall, truss,...) considered as isolated with appropriate boundary and support conditions.

DESIGN FIRE SCENARIO. Specific fire scenario on which an analysis will be conducted.

EXTERNAL FIRE CURVE. Nominal temperature-time curve intended for the outside of separating external walls which can be exposed to fire from different parts of the facade, i.e. directly from the inside of the respective fire compartment or from a compartment situated below or adjacent to the respective external wall.

FIRE LOAD DENSITY. Fire load per unit area related to the floor area q_f, or related to the surface area of the total enclosure, including openings, q_t.

FIRE LOAD. Sum of thermal energies which are released by combustion of all combustible materials in a space (building contents and construction elements).

HYDROCARBON FIRE CURVE. Nominal temperature-time curve for representing effects of an hydrocarbon type fire.

OPENING FACTOR. Factor representing the amount of ventilation depending on the area of openings in the compartment walls, on the height of these openings and on the total area of the enclosure surfaces.

STANDARD TEMPERATURE-TIME CURVE. Nominal curve defined in prEN 13501-2 for representing a model of a fully developed fire in a compartment.

TEMPERATURE-TIME CURVES. Gas temperature in the environment of member surfaces as a function of time. They may be:

- nominal: conventional curves, adopted for classification or verification of fire resistance, e.g. the standard temperature-time curve, external fire curve, hydrocarbon fire curve
- parametric: determined on the basis of fire models and the specific physical parameters defining the conditions in the fire compartment.

CONVECTIVE HEAT TRANSFER COEFFICIENT. Convective heat flux to the member related to the difference between the bulk temperature of gas bordering the relevant surface of the member and the temperature of that surface.

EMISSIVITY. Equal to absorptivity of a surface, i.e. the ratio between the radiative heat absorbed by a given surface and that of a black body surface.

FLASH-OVER. Simultaneous ignition of all the fire loads in a compartment.

3.3 Structural Fire design procedure

A structural fire design analysis should take into account the following steps as relevant:

- selection of the relevant design fire scenarios
- determination of the corresponding design fires
- calculation of temperature evolution within the structural members
- calculation of the mechanical behaviour of the structure exposed to fire.

Mechanical behaviour of a structure is depending on thermal actions and their thermal effect on material properties and indirect mechanical actions, as well as on the direct effect of mechanical actions.

Structural fire design involves applying actions for temperature analysis and actions for mechanical analysis according to this Part and other Parts of EN 1991. Actions on structures from fire exposure are classified as accidental actions, see EN 1990:2002, 6.4.3.3(4).

3.4 Design fire scenario, design fire

To identify the accidental design situation, the relevant design fire scenarios and the associated design fires should be determined on the basis of a fire risk assessment.

(2) For structures where particular risks of fire arise as a consequence of other accidental actions, this risk should be considered when determining the overall safety concept. Time- and load-dependent structural behaviour prior to the accidental situation needs not be considered, unless (2) applies.

For each design fire scenario, a design fire, in a fire compartment, should be estimated according to section 3 of this Part. The design fire should be applied only to one fire compartment of the building at a time, unless otherwise specified in the design fire scenario.

(3) For structures, where the national authorities specify structural fire resistance requirements, it may be assumed that the relevant design fire is given by the standard fire, unless specified otherwise.

3.5 Temperature Analysis

When performing temperature analysis of a member, the position of the design fire in relation to the member shall be taken into account. For external members, fire exposure through openings in facades and roofs should be considered.

(3) For separating external walls fire exposure from inside (from the respective fire compartment) and alternatively from outside (from other fire compartments) should be considered when required.

Depending on the design fire chosen in section 3, the following procedures should be used:

— with a nominal temperature-time curve, the temperature analysis of the structural members is made for a specified period of time, without any cooling phase;

Note The specified period of time may be given in the national regulations or obtained from annex F following the specifications of the national annex.

— with a fire model, the temperature analysis of the structural members is made for the full duration of the fire, including the cooling phase.

Note Limited periods of fire resistance may be set in the national annex.

3.6 Thermal actions for temperature analysis (Section 3)

Thermal actions are given by the net heat flux $\dot{h}_{net}[W/m^2]$ to the surface of the member. On the fire exposed surfaces the net heat flux $\dot{h}_{net}$ should be determined by considering heat transfer by convection and radiation as:

$$\dot{h}_{net} = \dot{h}_{net,c} + \dot{h}_{net,r} \qquad \textbf{(Eq. 3-1)}$$

where $\dot{h}_{net,c}$ is the net convective heat flux component and $\dot{h}_{net,r}$ is the net radiative heat flux component. The net convective heat flux component should be determined by:

$$\dot{h}_{net,c}\ [W/m^2] = \alpha_c \cdot (\theta_g - \theta_m) \qquad \textbf{(Eq. 3-2)}$$

where:

- α_c is the coefficient of heat transfer by convection $[W/m^2K]$
- θ_g is the gas temperature in the vicinity of the fire exposed member [°C]
- θ_m is the surface temperature of the member [°C].

On the unexposed side of separating members, the net heat flux $\dot{h}_{net}$ should be determined by using equation 3-1, with $\alpha_c = 4\ [W/m^2K]$. The coefficient of heat transfer by convection should be taken as $\alpha_c = 9\ [W/m^2K]$, when assuming it contains the effects of heat transfer by radiation.

The net radiative heat flux component per unit surface area is determined by:

$$\dot{h}_{net,r}\ [W/m^2] = \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot [(\theta_r + 273)^4 - (\theta_m + 273)^4] \qquad \textbf{(Eq. 3-3)}$$

where:

- Φ is the configuration factor
- ε_m is the surface emissivity of the member
- ε_f is the emissivity of the fire

- σ is the Stephan Boltzmann constant $(5,67 \times 10^{-8}\ W/m^2K^4)$
- θ_r is the effective radiation temperature of the fire environment [°C]
- θ_m is the surface temperature of the member [°C].

Note Unless given in the material related fire design Parts of prEN 1992 to prEN 1996 and prEN 1999, $\varepsilon_m = 0,8$ may be used. The emissivity of the fire is taken in general as $\varepsilon_f = 1,0$.

Where this Part or the fire design Parts of prEN 1992 to prEN 1996 and prEN 1999 give no specific data, the configuration factor should be taken as $\Phi = 1$. A lower value may be chosen to take account of so called position and shadow effects.

Note For the calculation of the configuration factor Φ a method is given in annex G.

In case of fully fire engulfed members, the radiation temperature θ_r may be represented by the gas temperature θ_g around that member. The surface temperature θ_m results from the temperature analysis of the member according to the fire design Parts 1-2 of prEN 1992 to prEN 1996 and prEN 1999, as relevant.

Gas temperatures θ_g may be adopted as nominal temperature-time curves according to 3.2, or adopted according to the fire models given in 3.3.

Note The use of the nominal temperature-time curves according to 3.2 or, as an alternative, the use of the natural fire models according to 3.3 may be specified in the national annex.

3.7 Nominal temperature-time curves

STANDARD TEMPERATURE-TIME CURVE. The standard temperature-time curve is given by:

$$\theta_g = 20 + 345 \cdot \log_{10}(8t + 1) \quad \text{(Eq. 3-4)}$$

where:

- θ_g is the gas temperature in the fire compartment [°C]
- t is the time [min].

The coefficient of heat transfer by convection is $\alpha_c = 25\ W/m^2K$.

EXTERNAL FIRE CURVE. The external fire curve is given by:

$$\theta_g = 20 + 660(1 - 0,687e^{-0,32t} - 0,313e^{-3,8t}) \quad \text{(Eq. 3-5)}$$

where:

- θ_g is the gas temperature near the member [°C]
- t is the time [min].

$$\dot{h}_{net,c} = \alpha_c \cdot (\theta_g - \theta_m) = 4,00 \cdot (720 - 500) = 880\ \mathrm{W/m^2} = 0,88\ \mathrm{kW/m^2}$$

$$\dot{h}_{net,r} = \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot [(\theta_r + 273)^4 - (\theta_m + 273)^4] = 1 \cdot 0,8 \cdot 1 \cdot \frac{5,67}{10^8} \cdot [(720 + 273)^4 - (500 + 273)^4]$$

$$\dot{h}_{net,r} = 1 \cdot 0,8 \cdot 1 \cdot \frac{5,67}{10^8} \cdot [9,723 \cdot 10^{11} - 3,570 \cdot 10^{11}] = 27,91 \cdot \frac{10^{11}}{10^8} = 27910\ \mathrm{W/m^2} = 27,91\ \mathrm{kW/m^2}$$

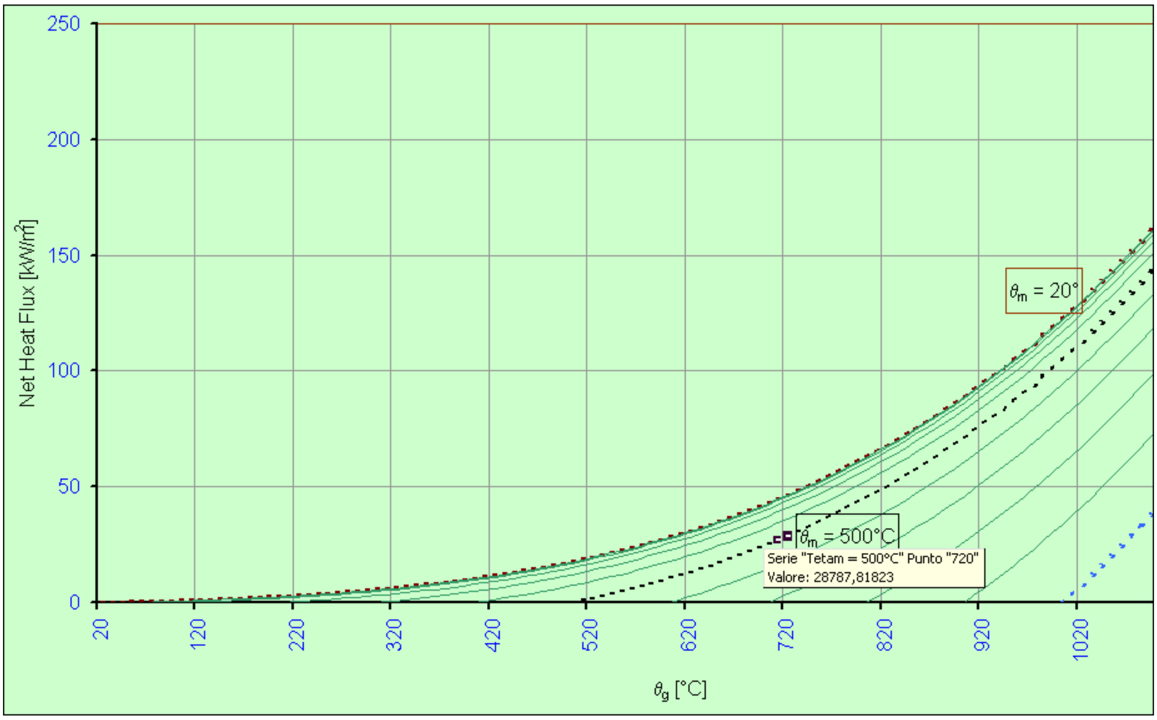


Figure 3.6 View Plot (from input). See cells Range H63:J65 - Sheet: CodeSec3.

Hence, we find: $\dot{h}_{net} = \dot{h}_{net,c} + \dot{h}_{net,r} = 0,88 + 27,91 = 28,79\ \mathrm{kW/m^2}$ (see plot above).

example-end

EXAMPLE 3-S- Section 3.2 - Nominal temperature-time curves - **test 2**

Given: Determine the standard temperature-time curve at $t = 120$ min (time of the exposure), the external fire curve and the hydrocarbon temperature-time curve at $t = 15$ min.

[*Reference sheet:* CodeSec3]-[*Cell-Range: A68:O68-A190:O190*].

Solution: The standard temperature-time curve is given by (gas temperature in the fire compartment): $\theta_g = 20 + 345 \cdot \log_{10}(8t + 1)$.

Sobstituting $t = 120$ min, we get:

$$\theta_g = 20 + 345 \cdot \log_{10}(8t + 1) = 20 + 345 \cdot \log_{10}(8 \cdot 120 + 1) = 20 + 345 \cdot 2,983 = 1049°C .$$

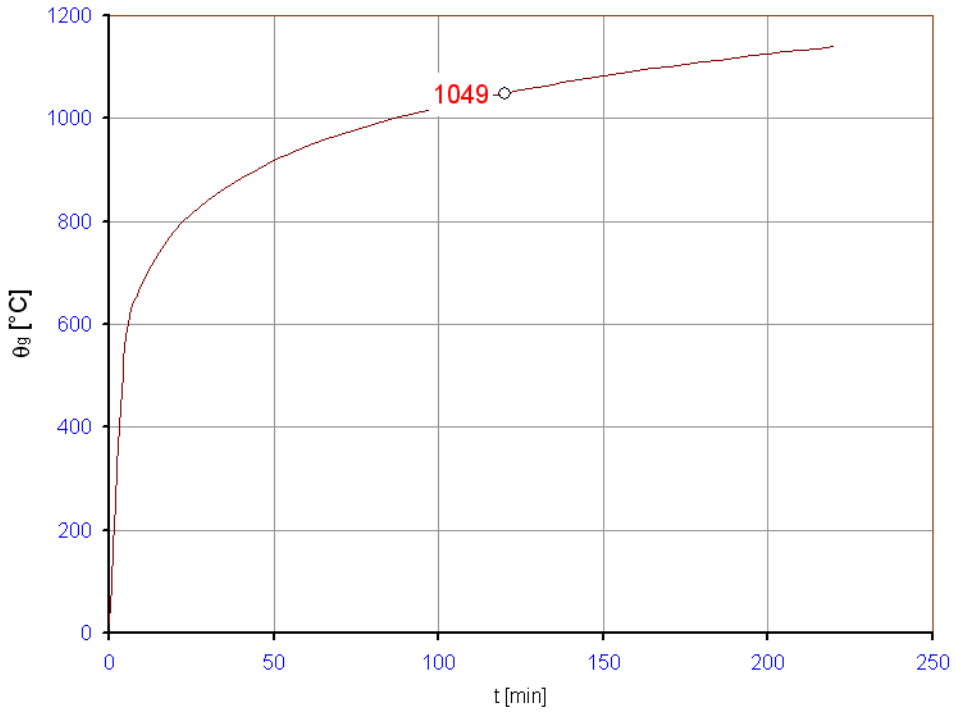


Figure 3.7 Standard temperature-time curve.

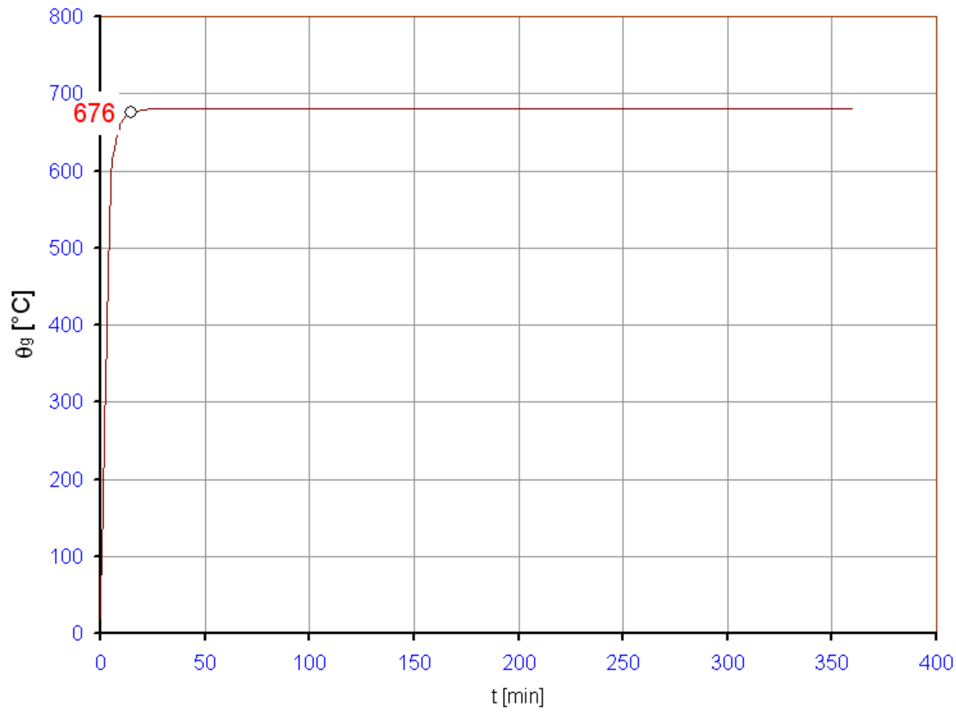


Figure 3.8 External fire curve.

The external fire curve is given by (gas temperature near the member):

$\theta_g = 20 + 660(1 - 0,687e^{-0,32t} - 0,313e^{-3,8t})$. Sobstituting $t = 15\ \text{min}$, we get:

$\theta_g = 20 + 660(1 - 0,687e^{-0,32(15)} - 0,313e^{-3,8(15)}) = 20 + 660(1 - 0,687e^{-4,8} - 0,313e^{-57})$

$\theta_g \approx 20 + 660(1 - 0,687 \cdot 0,00823 - 0) = 676,3°C$.

We find that: $\theta_g = 680°C = cost$ for $t > 40\ \text{min}$ approximately.

The hydrocarbon temperature-time curve is given by (gas temperature in the fire compartment): $\theta_g = 20 + 1080(1 - 0,325e^{-0,167t} - 0,675e^{-2,5t})$.

Sobstituting $t = 15\ \text{min}$, we get:

$\theta_g = 20 + 1080(1 - 0,325e^{-0,167(15)} - 0,675e^{-2,5(15)}) = 20 + 1080(1 - 0,325e^{-2,505} - 0,675e^{-(37,5)})$

$\theta_g \approx 20 + 1080(1 - 0,325e^{-2,505} - 0) = 20 + 1080(1 - 0,325 \cdot 0,0817 - 0) = 1071,3°C$.

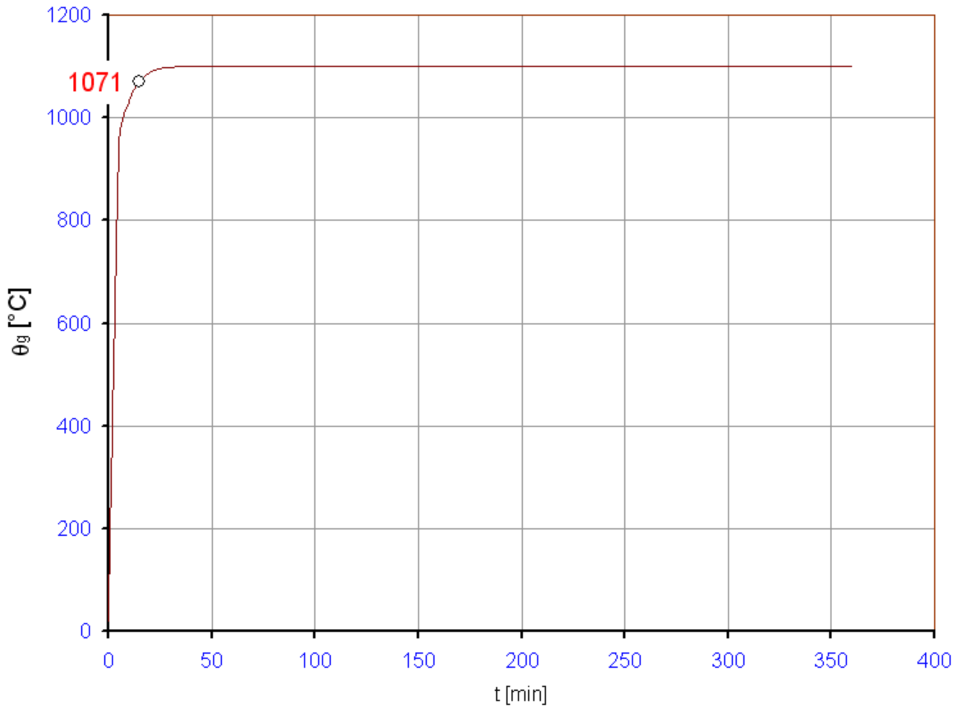


Figure 3.9 Hydrocarbon curve.

We find that: $\theta_g = 1100°C = cost$ for $t > 65\ \text{min}$ approximately.

example-end

EXAMPLE 3-T- Annex A - Parametric temperature-time curves - **test 3**

Given: For internal members of fire compartments, calculate the gas temperature in the compartment using the method given in informative Annex A of EC1 Part 1-2. The theory assumes that temperature rise is independent of fire load.

The temperature within the compartment is assumed to vary as a simple exponential function of modified time dependent on the variation in the ventilation area and the properties of the compartment linings from this "standard" compartment.

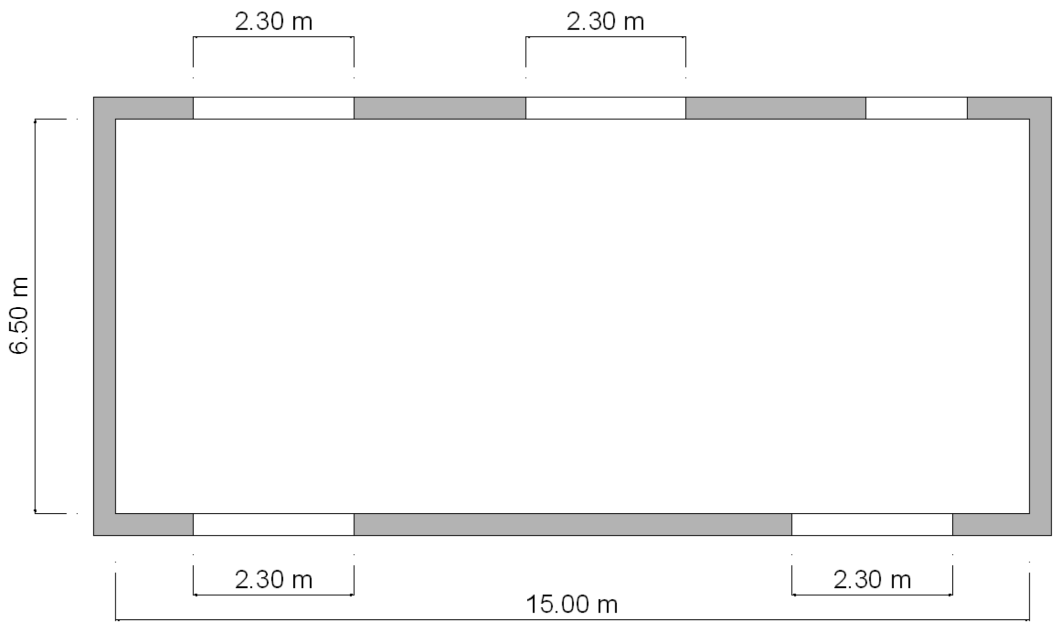


Figure 3.10 Plan of fire compartment (height = 3,60 m).

[*Reference sheet:* Annex A]-[*Cell-Range: A1:O1-A152:O152*].

Solution: Dimension of the compartment:

width = 6,50 m; lenght = 15,00 m; heigth = 3,60 m.

Dimension of windows:

number of windows = 4; width = 2,30 m (mean value); heigth = h_{eq} = 1,70 m (weighted average of window heights on all walls).

	ρ [kg/m^3]	c [J/kgK]	λ [W/mK]	$b = \sqrt{\rho c \lambda}$ [J/m^{2}s^{0,5}K]
CEILING	2400	1506	1,50	$\sqrt{2400 \cdot 1506 \cdot 1,50} = 2328$ (a)
WALLS	900	1250	0,24	$\sqrt{900 \cdot 1250 \cdot 0,24} = 519,6$
FLOOR	900	1250	0,24	$519,6$

Table 3.5 Thermal properties of enclosure surfaces.

(a). b (thermal absorptivity) with the following limits $100 \le b \le 2200$.

We assume: ceiling $b = 2200\ \mathrm{J/m^2 s^{0,5} K}$; walls and floor $b = 520\ \mathrm{J/m^2 s^{0,5} K}$.

Total area of vertical openings on all walls:

$A_v = 4 \cdot (2,30\ m) \cdot (1,70\ m) = 15,64\ m^2$.

Total area of enclosure (walls, ceiling and floor, including openings):

$A_t = 2 \cdot [(6,50 \cdot 15,00) + (6,50 + 15,00) \cdot 3,60] = 349,8\ m^2$.

Opening factor:

$$O = A_v \frac{\sqrt{h_{eq}}}{A_t} = (15,64)\frac{\sqrt{(1,70)}}{(349,8)} = 0,0583\ m^{1/2}$$

with the following limits: $0,02 \le O = 0,0583 \le 0,20$.

We find: ceiling $A_j = 6,50 \cdot 15,00 = 97,50\ m^2$ and floor $A_j = 97,50\ m^2$, wall $A_j = 2 \cdot (6,50 + 15,00) \cdot 3,60 - 15,64 = 139,2\ m^2$. Hence, we get:

$$b = \frac{\sum b_j A_j}{(A_t - A_v)} = \frac{2200 \cdot 97,50 + 520 \cdot 139,2 + 520 \cdot 97,5}{(349,8 - 15,64)} = 1010\ J/m^2 s^{0,5} K.$$

with the following limits: $100 \le b = 1010 \le 2200$.

Time factor function:

$$\Gamma = \frac{[O/b]^2}{(0,04/1160)^2} = \frac{[(0,0583)/1010]^2}{(0,04/1160)^2} = 2,802.$$

Design value of the fire load density related to the surface area A_f of the floor:

$q_{f,d} = 700\ MJ/m^2$.

Floor area of the fire compartment: $A_f = 97,5\ m^2$.

Design value of the fire load density related to the total surface area A_t of the enclosure:

$$q_{t,d} = q_{f,d}\frac{A_f}{A_t} = 700 \cdot \frac{97,5}{349,8} = 195,11\ MJ/m^2.$$

Fire growth rate: say $t_{lim} = 20\ min \approx 0,333\ h$ (medium fire growth rate).

$0,2 \cdot 10^{-3} q_{t,d}/O = (0,2 \cdot 10^{-3} \cdot 195,11)/0,0583 = 0,67\ h$.

$t_{max} = max[0,2 \cdot 10^{-3} q_{t,d}/O; 0,333\ h] = max[0,67; 0,333] = 0,67\ h$.

$t_{max} > t_{lim}$ the fire is ventilation controlled.

The maximum temperature θ_{max} in the heating phase happens for $t^* = t^*_{max}$:

$t^*_{max} = t_{max} \cdot \Gamma = 0,67 \cdot 2,802 = 1,88\ h$.

Maximum temperature (heating phase):

$$\begin{cases} \theta_{max} = 20 + 1325 \cdot (1 - 0,324e^{-0,2t^*} - 0,204e^{-1,7t^*} - 0,472e^{-19t^*}) \\ t^*_{max} = 1,88\ h \end{cases}$$

$\theta_{max} = 20 + 1325 \cdot (1 - 0,324e^{-0,2(1,88)} - 0,204e^{-1,7(1,88)} - 0,472e^{-19(1,88)})$

$\theta_{max} \approx 20 + 1325 \cdot [1 - 0,324 \cdot (0,687) - 0,204 \cdot (0,041) - 0] = 1039°C$.

Cooling phase $t \ge t^*_{max}$:

with $t_{max} = 0,67\ h > t_{lim} = 0,33\ h$, we get: $x = 1$ (see eq. A.12).

With $t^{**}_{max} = \left(0,2\times10^{-3}\cdot\frac{q_{t,d}}{O}\right)\cdot\Gamma = \left(0,2\times10^{-3}\cdot\frac{195,11}{0,0583}\right)\cdot 2,802 = 0,669\cdot 2,802 = 1,88\ h$

$0,5\ h < \left(0,2\times10^{-3}\cdot\frac{q_{t,d}}{O}\right)\cdot\Gamma = 1,88\ h < 2\ h$, we get:

$\theta_g = \theta_{max} - 250\cdot(3 - t^{**}_{max})\cdot(t^{*} - t^{**}_{max}\cdot x)$,

$\theta_g = \theta_{max} - 250\cdot(3-1,88)\cdot(t^{*}-1,88) = 1039 - 250\cdot(3-1,88)\cdot(t^{*}-1,88)$.

For (say) $t = 1,10\ h \Rightarrow t^{*} = t\cdot\Gamma = 1,10\cdot 2,802 = 3,08\ h$, we find:

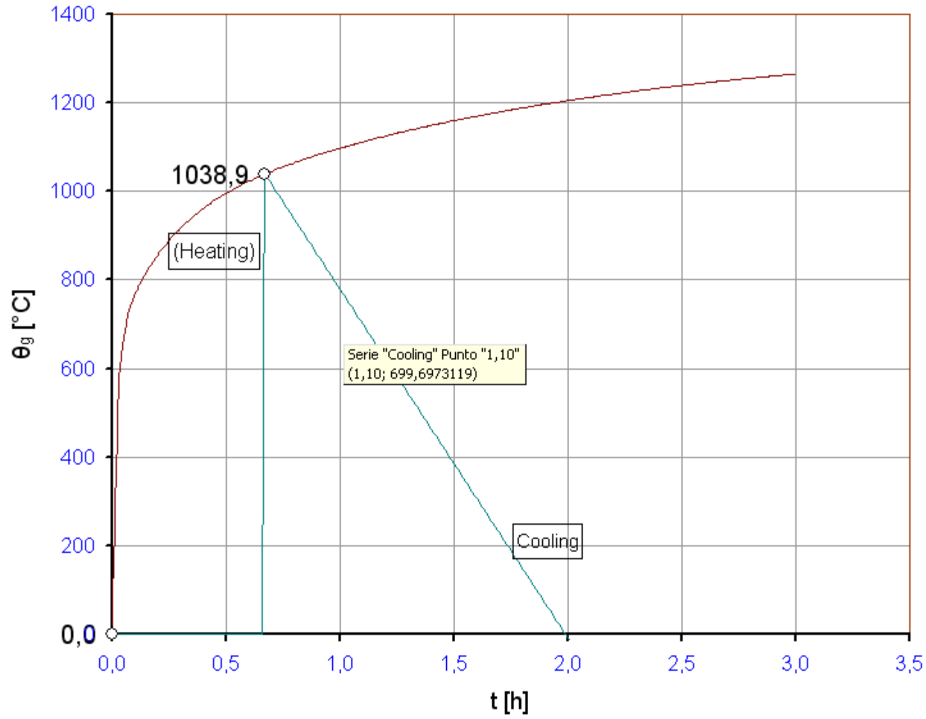


Figure 3.11 Parametric curve: heating, cooling.

$\theta_g = 1039 - 250\cdot(3-1,88)\cdot(t^{*}-1,88) = 1039 - 250\cdot(3-1,88)\cdot(3,08-1,88) = 703°C$.

Rounding error: 100 x (703 – 699,7)/699,7 = 0,5%.

example-end

EXAMPLE 3-U-- Annex A - Parametric temperature-time curves - **test 4**

Given: Maintaining the same assumptions in the previous example and assuming $q_{f,d} = 200\ MJ/m^2$, calculate the cooling phase.

[*Reference sheet:* Annex A]-[*Cell-Range: A107:O107-A152:O152*].

Solution: We find:

$$q_{t,d} = q_{f,d}\frac{A_f}{A_t} = 200 \cdot \frac{97,5}{349,8} = 55,75\ MJ/m^2$$

$$t_{max} = \max[0,2 \cdot 10^{-3} q_{t,d}/O;\ 0,333\ h] = \max[0,19;\ 0,333] = 0,333\ h$$

Time factor function (A.2b):

$$\Gamma_{lim} = \frac{[O_{lim}/b]^2}{(0,04/1160)^2} \approx \frac{[(0,0167)/1010]^2}{(0,04/1160)^2} = 0,23\text{, with}$$

$$O_{lim} = \frac{0,1}{10^3}\frac{q_{t,d}}{t_{lim}} = \frac{0,1}{10^3}\frac{(55,75)}{(0,333)} = 0,0167\,.$$

If ($O > 0{,}04$ and $q_{t,d} < 75$ and $b < 1160$), Γ_{lim} in (A.8) has to be multiplied by k given by:

$$k = 1 + \left(\frac{O - 0,04}{0,04}\right) \cdot \left(\frac{q_{t,d} - 75}{75}\right) \cdot \left(\frac{1160 - b}{1160}\right) = 1 + \left(\frac{0,0583 - 0,04}{0,04}\right) \cdot \left(\frac{55,75 - 75}{75}\right) \cdot \left(\frac{1160 - 1010}{1160}\right)$$

$$k = 1 + 0,4575 \cdot (-0,2567) \cdot (0,1293) = 0,98\,.$$

We get:

$$t^*_{max} = t_{max} \cdot k\Gamma_{lim} = 0,333 \cdot 0,98 \cdot 0,231 \approx 0,08\ h\,.$$

Maximum temperature (heating phase):

$$\begin{cases} \theta_{max} = 20 + 1325 \cdot (1 - 0,324e^{-0,2t^*} - 0,204e^{-1,7t^*} - 0,472e^{-19t^*}) \\ t^*_{max} \approx 0,757\ h \end{cases}$$

$$\theta_{max} = 20 + 1325 \cdot (1 - 0,324e^{-0,2(0,076)} - 0,204e^{-1,7(0,076)} - 0,472e^{-19(0,076)})$$

$$\theta_{max} \approx 20 + 1325 \cdot [1 - 0,324 \cdot (0,985) - 0,204 \cdot (0,879) - 0,472 \cdot (0,236)] = 537°C\,,$$

Rounding error: 100 x (537 – 536,1)/536,1 < 0,2%.

$$t^{**}_{max} = \frac{0,2}{10^3}\frac{q_{t,d}}{O}\Gamma = \frac{0,2}{10^3}\frac{(55,75)}{(0,0583)} \cdot 2,802 = 0,536\ h\,.$$

$t_{max} \le t_{lim} = 0,333\ h$ (the fire is fuel controlled):

$$x = \frac{t_{lim} \cdot \Gamma}{t^{**}_{max}} \approx \frac{(0,333) \cdot 2,802}{0,536} = 1,74\,.$$

For $0,5 < t^{**}_{max} < 2$:

$$\theta_g = \theta_{max} - 250 \cdot (3 - t^{**}_{max}) \cdot (t^* - t^{**}_{max} \cdot x) = \theta_{max} - 250 \cdot (3 - 0,536) \cdot (t^* - 0,536 \cdot 1,74)\,.$$

For (say) $t = 0,50\ h \Rightarrow t^* = t \cdot \Gamma = 0,50 \cdot 2,802 = 1,40\ h$, we find:

$$\theta_g \approx 537 - 250 \cdot (3 - 0,536) \cdot (1,40 - 0,536 \cdot 1,74) = 249°C\,,$$

Rounding error: 100 x (249 – 248,4)/248,4 < 0,25%.

example-end

3.9 References [Section 3]

BS EN 1991-1-2. Eurocode 1: *Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*. 26 November 2002

EN 1991-1-2:2002/AC:2013. Eurocode 1: Actions on structures - Part 1-2: General actions - Actions on structures exposed to fire. CEN Brussels, February 2013.

Manual for the design of building structures to Eurocode 1 and Basis of Structural Design - April 2010. © 2010 The Institution of Structural Engineers.

Section 4 **Eurocode 1**
EN 1991-1-2
Annex B

4.1 Thermal actions for external members - Simplified calculation method

This method considers steady-state conditions for the various parameters. The method is valid only for fire loads $q_{f,d} > 200\ MJ/m^2$. This method allows the determination of:

— the maximum temperatures of a compartment fire

— the size and temperatures of the flame from openings

— radiation and convection parameters.

CONDITIONS OF USE. When there is more than one window in the relevant fire compartment, the weighted average height of windows h_{eq}, the total area of vertical openings A_v and the sum of windows widths are used.

When there are windows in only wall 1, the ratio D/W is given by:

$$D/W = W_2/W_1 . \qquad \text{(Eq. 4-7)}$$

When there are windows on more than one wall, the ratio D/W has to be obtained as follows:

$$D/W = \frac{W_2}{W_1}\frac{A_{v1}}{A_v} , \qquad \text{(Eq. 4-8)}$$

where:

- W_1 is the width of the wall 1, assumed to contain the greatest window area
- A_{v1} is the sum of window areas on wall 1
- W_2 is the width of the wall perpendicular to wall 1 in the fire compartment.

When there is a core in the fire compartment, the ratio D/W has to be obtained as follows:

$$D/W = \frac{(W_2 - L_c)A_{v1}}{(W_1 - W_c)A_v}, \qquad \textbf{(Eq. 4-9)}$$

where:

- L_c and W_c are the length and width of the core
- W_1 and W_2 are the length and width of the 'fire compartment
- the size of the fire compartment should not exceed 70 m in length, 18 m in width and 5 m in height.

(5) All parts of an external wall that do not have the fire resistance (REI) required for the stability of the building should be classified as window areas. The total area of windows in an external wall is:

— the total area, according to (5), if it is less than 50% of the area of the relevant external wall of the compartment

— firstly the total area and secondly 50% of the area of the relevant external wall of the compartment if, according to (5), the area is more than 50%. These two situations should be considered for calculation. When using 50% of the area of the external wall, the location and geometry of the open surfaces should be chosen so that the most severe case is considered.

The flame temperature should be taken as uniform across the width and the thickness of the flame.

EFFECT OF WIND - MODE OF VENTILATION, DEFLECTION BY WIND. If there are windows on opposite sides of the fire compartment or if additional air is being fed to the fire from another source (other than windows), the calculation shall be done with forced draught conditions. Otherwise, the calculation is done with no forced draught conditions.

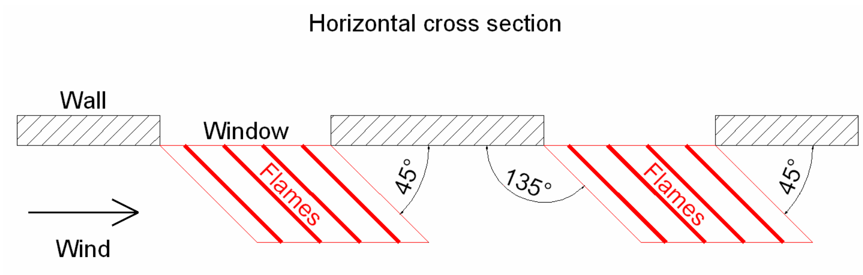


Figure 4.12 Deflection of flame by wind (from fig. B.1).

Flames from an opening should be assumed to be leaving the fire compartment (see figure below):

— perpendicular to the facade

— with a deflection of 45° due to wind effects.

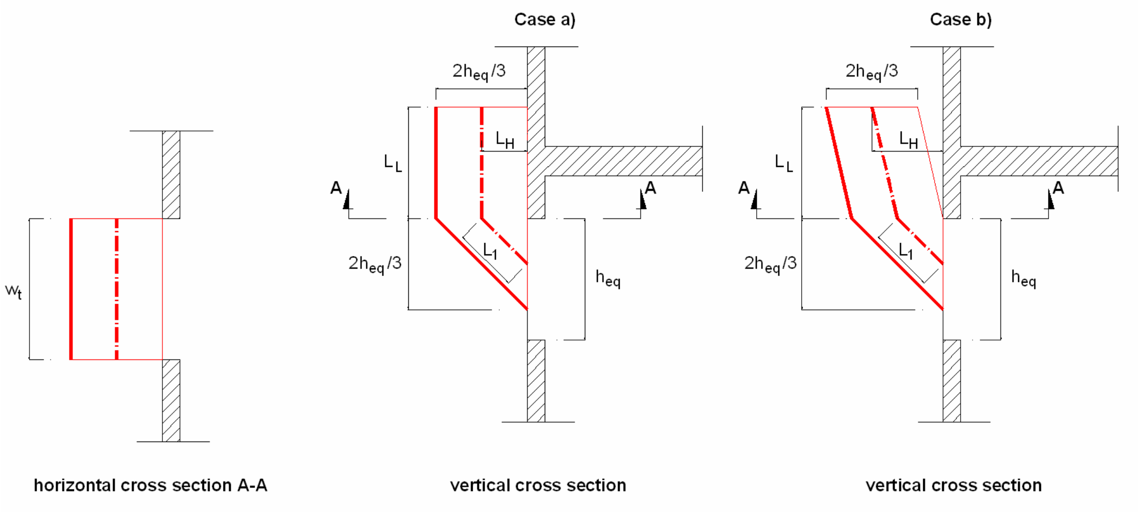


Figure 4.13 Flame dimensions, no through draught (from fig. B.2).

CHARACTERISTIC OF FIRE AND FLAMES: NO FORCED DRAUGHT. The rate of burning or the rate of heat release is given by [MW]:

$$Q = \min\left\{\left(\frac{A_f \cdot q_{f,d}}{\tau_F}\right);\ 3,15 \cdot \left[1 - e^{-\frac{0,036}{O}}\right] \cdot A_v \cdot \left(\frac{h_{eq}}{D/W}\right)^{1/2}\right\}. \quad \textbf{(Eq. 4-10)}$$

The temperature of the fire compartment is given by [°K]:

$$T_f = 6000 \cdot (1 - e^{-0,1/O}) \cdot \sqrt{O} \cdot (1 - e^{-0,00286\Omega}) + T_0 . \quad \textbf{(Eq. 4-11)}$$

The flame height (see Figure B.2) is given by:

$$L_L = \max\left\{0;\ h_{eq} \cdot \left[2,37 \cdot \left(\frac{Q}{A_v \rho_g \sqrt{h_{eq} g}}\right)^{2/3} - 1\right]\right\} \quad \textbf{(Eq. 4-12)}$$

where:

- A_v is the total area of vertical openings on all walls $A_v = \sum_i A_{v,i}$
- $h_{eq} = \left(\sum_i A_{v,i} h_i\right)/A_v$ is the weighted average of windows on all walls
- A_t is the total area of enclosure (walls, ceiling and floor, including openings)
- $q_{f,d}$ is the design fire load density $[MJ/m^2]$ related to the floor area A_f
- A_f is the floor area of the fire compartment
- $O = A_v \cdot (\sqrt{h_{eq}}/A_t)$ is the "opening factor" of the fire compartment
- $\tau_F = 1200\ s$ is the free burning duration (in seconds)

- D/W i the "ratio" (see section B.2 "*Conditions of use*")
- $\Omega = A_f q_{f,d}/\sqrt{A_v A_t}$
- $T_0 = 273\ °K = 20°C$ is the "initial temperature"
- ρ_g is the internal gas density $[kg/m^3]$
- $g = 9,81\ m/s^2$.

The flame width is the window width (see Figure B.2). The flame depth is 2/3 of the window height: 2/3 h_{eq} (see Figure B.2).

(6) The horizontal projection of flames:

— in case of a wall existing above the window, is given by: $L_H = h_{eq}/3$ if $h_{eq} \le 1,25 w_t$; $L_H = 0,3 h_{eq}(h_{eq}/w_t)^{0,54}$ if $h_{eq} > 1,25 w_t$ and distance to any other window > $4w_t$; $L_H = 0,454 h_{eq}(h_{eq}/2w_t)^{0,54}$ in other cases, (with w_t = sum of window widths on all walls)

— in case of a wall not existing above the window, is given by: $L_H = 0,6 h_{eq}(L_L/h_{eq})^{1/3}$.

The flame length along axis is given by:

— when $L_L > 0$, $L_f = L_L + h_{eq}/2$ if wall exists above window or if $h_{eq} \le 1,25 w_t$; $L_f = \sqrt{L_L^2 + (L_H - h_{eq}/3)^2} + h_{eq}/2$ if no wall exists above window or if $h_{eq} > 1,25 w_t$

— when $L_L = 0$, then $L_f = 0$.

The flame temperature at the window is given by [°K]:

$$T_w = \frac{520}{1 - \left[0,4725 \cdot \left(\frac{L_f \cdot w_t}{Q}\right)\right]} + T_0 \qquad \textbf{(Eq. 4-13)}$$

with $L_f w_t/Q < 1$. The emissivity of flames at the window may be taken as $\varepsilon_f = 1,0$. The flame temperature along the axis is given by [°K]:

$$T_z = (T_w - T_0) \cdot \left\{1 - \left[0,4725 \cdot \left(\frac{L_x \cdot w_t}{Q}\right)\right]\right\} + T_0 \qquad \textbf{(Eq. 4-14)}$$

with $L_x w_t/Q < 1$ and L_x is the axis length from the window to the point where the calculation is made. The emissivity of flames may be taken as:

$$\varepsilon_f = 1 - e^{-0,3 d_f} \qquad \textbf{(Eq. 4-15)}$$

where d_f is the flame thickness [m]. The convective heat transfer coefficient is given by $[W/m^2K]$:

$$\alpha_c = 4,67 \cdot (1/d_{eq})^{0,4} \cdot (Q/A_v)^{0,6}. \qquad \textbf{(Eq. 4-16)}$$

(13) If an awning or balcony is located at the level of the top of the window on its whole width for the wall above the window and $h_{eq} \leq 1,25w_t$, the height and horizontal projection of the flame should be modified as follows:

— the flame height L_L given in eq. 4-12 is decreased by $W_a(1+\sqrt{2})$

— the horizontal projection of the flame given in (6), is increased by W_a.

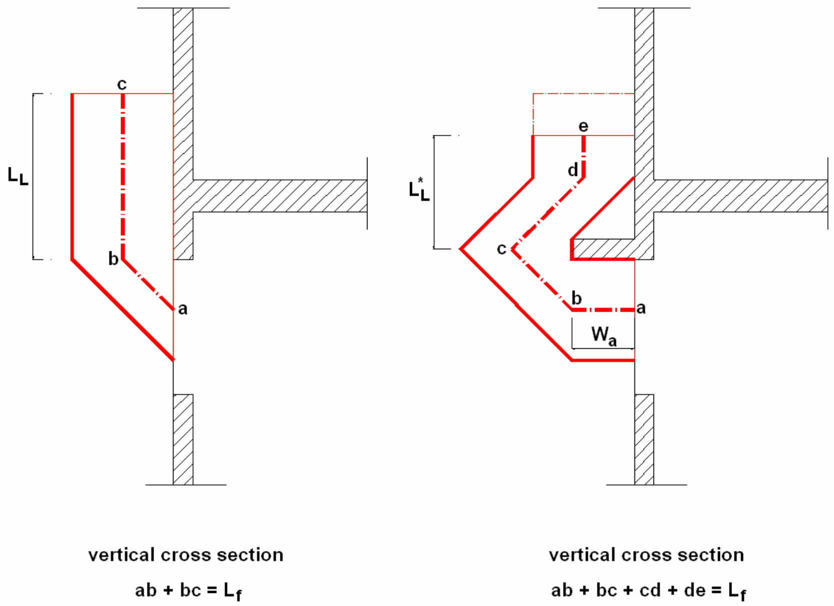


Figure 4.14 Deflection of flame by balcony (from fig. B.3).

With the same conditions for awning or balcony as mentioned in (13), in the case of no wall above the window or $h_{eq} > 1,25w_t$, the height and horizontal projection of the flame should be modified as follows:

— the flame height L_L given in eq. 4-12 is decreased by W_a

— the horizontal projection of the flame L_H, obtained in (6) with the above mentioned value of L_L, is increased by W_a.

FORCED DRAUGHT. The rate of burning or the rate of heat release is given by [MW]:

$$Q = \left(\frac{A_f \cdot q_{f,d}}{\tau_F}\right). \qquad \textbf{(Eq. 4-17)}$$

The temperature of the fire compartment is given by [°K]:(1)

$$T_f = 1200 \cdot (1 - e^{-0,00288\Omega}) + T_0. \qquad \textbf{(Eq. 4-18)}$$

(1) There were errors in the equation B.19 of Annex B of the English version of the standard. These have been corrected in the BS EN 1991-1-2:2002.

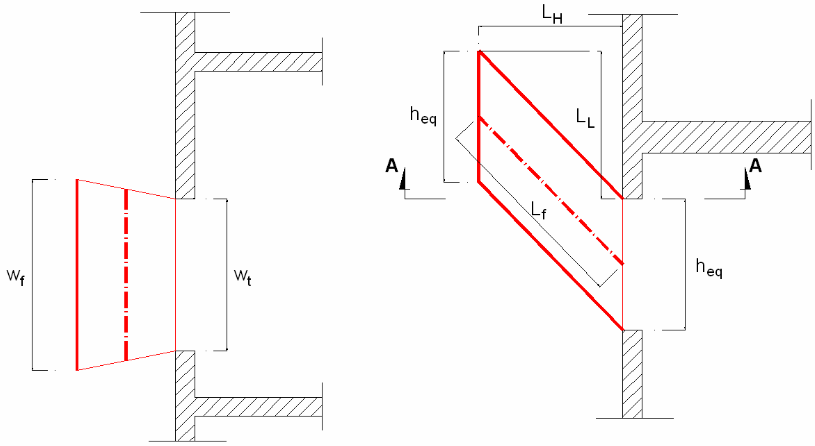


Figure 4.15 Flame dimensions, through or forced draught (from fig. B.4).

The flame height (see Figure B.4) is given by:

$$L_L = \left(1,366 \cdot (1/u)^{0,43} \cdot \frac{Q}{\sqrt{A_v}}\right) - h_{eq} \quad \textbf{(Eq. 4-19)}$$

where $u[m/s]$ is the wind speed, moisture content. The horizontal projection of flames is given by:

$$L_H = 0,605 \cdot (u^2/h_{eq})^{0,22} \cdot (L_L + h_{eq}) . \quad \textbf{(Eq. 4-20)}$$

The flame width is given by $w_f = w_t + 0,4L_H$. The flame length along axis is given by $L_f = (L_L^2 + L_H^2)^{0,5}$.

The flame temperature at the window is given by [°K]:

$$T_w = \frac{520}{1 - \left[0,3325 \cdot \left(\frac{L_f \cdot \sqrt{A_v}}{Q}\right)\right]} + T_0 \quad \textbf{(Eq. 4-21)}$$

with $L_f\sqrt{A_v}/Q < 1$. The emissivity of flames at the window may be taken as $\varepsilon_f = 1,0$. The flame temperature along the axis is given by [°K]:

$$T_z = (T_w - T_0) \cdot \left\{1 - \left[0,3325 \cdot \left(\frac{L_x \cdot \sqrt{A_v}}{Q}\right)\right]\right\} + T_0 \quad \textbf{(Eq. 4-22)}$$

where L_x is the axis length from the window to the point where the calculation is made. The emissivity of flames may be taken as:

$$\varepsilon_f = 1 - e^{-0,3d_f} \quad \textbf{(Eq. 4-23)}$$

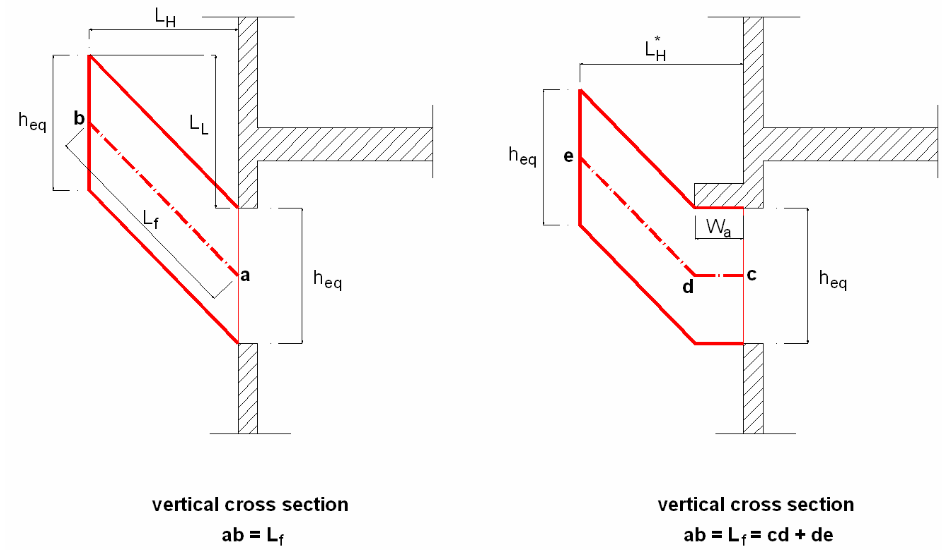


Figure 4.16 Deflection of flame by awning (from fig. B.5).

where d_f is the flame thickness [m]. The convective heat transfer coefficient is given by [W/m^{2}K]:

$$\alpha_c = 9,8 \cdot (1/d_{eq})^{0,4} \cdot \left(\frac{Q}{17,5 A_v} + \frac{u}{1,6}\right)^{0,6}. \quad \textbf{(Eq. 4-24)}$$

Regarding the effects of balconies or awnings, see Figure B.5 (below), the flame trajectory, after being deflected horizontally by a balcony or awning, is the same as before, i.e. displaced outwards by the depth of the balcony, but with a flame length L_f unchanged.

4.2 Verification tests

EN1991-1-2_(B).XLS. 6.85 MB. Created: 6 February 2013. Last/Rel.-date: 3 June 2013. Sheets:

— Splash

— Annex B.

EXAMPLE 4-V-Section B.2 - Conditions of use - **test 1**

Given: Find the ratio D/W when:

Case 1) there are windows in only one wall

Case 2) there are windows on more than one wall

Case 3) there is a core in the fire compartment.

When Case 1) or 2) applies, assume that:

– the width W_1 of the wall 1 (assumed to contain the greatest window area) is equal to 0,40 m

– the width W_2 of the wall perpendicular to wall 1 in the fire compartment is equal to 0,25 m

– the sum A_{v1} of windows areas on wall 1 is equal to 4,20 m^2

– the total area A_v of vertical openings on all walls is equal to 6,80 m^2.

When Case 3) applies, assume that:

– the length L_c and width W_c of the core are equal to 5,00 m and 3,50 m respectively

– the length W_1 and the width W_2 of the fire compartment are equal to 6,00 m and 6,50 m respectively.

[*Reference sheet:* Annex B]-[*Cell-Range: A75:Q75-CommandButton*].

Solution: Case 1). From eq. (B.1):

$$D/W = \frac{W_2}{W_1} = \frac{0,25}{0,40} = 0,625\,.$$

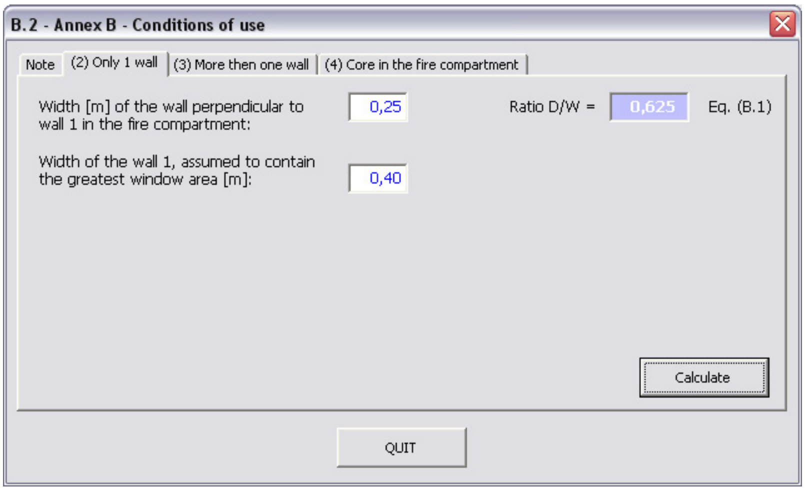


Figure 4.17 PreCalculus Excel® form: procedure for a quick pre-calculation: Case 1).

Case 2). From eq. (B.2):

$$D/W = \frac{W_2 A_{v1}}{W_1\, A_v} = \frac{0,25(4,20)}{0,40(6,80)} = 0,386\,.$$

Case 3). From eq. (B.3):

$$D/W = \frac{(W_2 - L_c)A_{v1}}{(W_1 - W_c)A_v} = \frac{(6,50 - 5,00)(4,20)}{(6,00 - 3,50)(6,80)} = 0,371\,.$$

Note All parts of an external wall that do not have the fire resistance (REI) required for the stability of the building should be classified as window areas.

Using the PreCalculus Excel form, for the Case 2) we find:

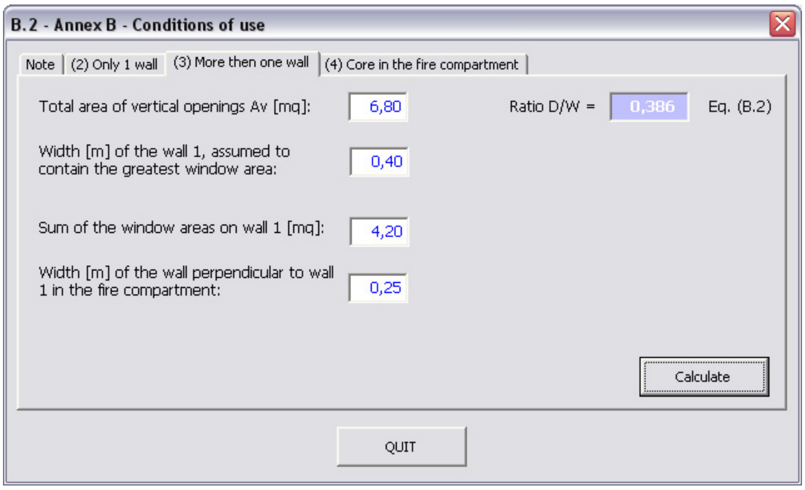


Figure 4.18 PreCalculus Excel form: procedure for a quick pre-calculation: Case 2).

Finally, for the case 3), we find:

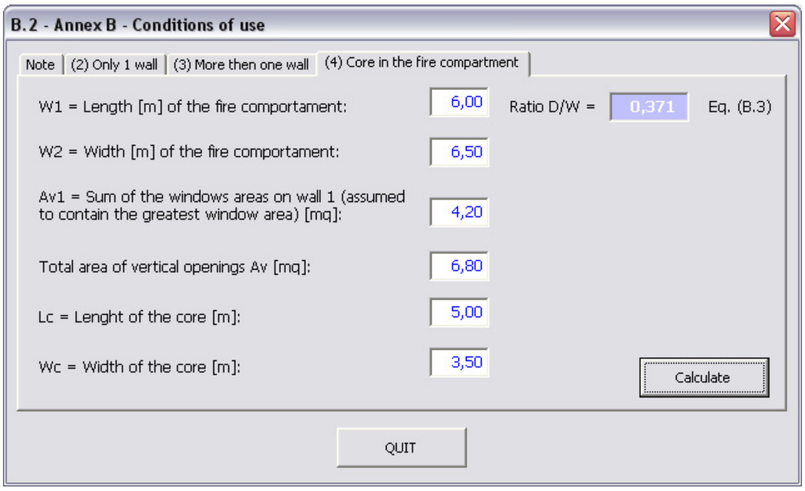


Figure 4.19 PreCalculus Excel form: procedure for a quick pre-calculation: Case 3).

Note The size of the fire compartment should not exceed 70 m in length, 18 m in width and 5 m in height. The flame temperature should be taken as uniform across the width and the thickness of the flame.

example-end

EXAMPLE 4-W- Sec. B.4.1, Characteristic of fire and flames: no awning or balcony - **test2**

Given: Assume that: $A_f = 30,00\ m^2$; $q_{f,d} = 500\ MJ/m^2$ (taken from Annex E); $h_{eq} = 1,70\ m$; $A_t = 112,00\ m^2$; $A_v = 6,80\ m^2$ and D/W = 0,625.

Assuming no awning or balcony is located at the level of the top of the windows, find:

– the rate of burning

– the temperature of the fire compartment

– the flame height, width and depth

– the horizontal projection of flames

– the flame temperature at the window

– the flame temperature along the axis

– the emissivity of flames

– the convective heat transfer coefficient.

[*Reference sheet:* Annex B]-[*Cell-Range: A59:O59-A264:O264 and A343:O343-A502:O502*].

Solution: **No forced draught.**

"Opening factor" of the fire compartment:

$O = A_v \cdot (\sqrt{h_{eq}}/A_t) = 6,80 \cdot (\sqrt{1,70}/112) = 0,0792\ m^{1/2}$.

Rate of burning or rate of heat release (eq. 4-10):

$$Q = \min\left\{\left(\frac{A_f \cdot q_{f,d}}{\tau_F}\right);\ 3,15 \cdot \left[1 - e^{-\frac{0,036}{O}}\right] \cdot A_v \cdot \left(\frac{h_{eq}}{D/W}\right)^{1/2}\right\}$$

$$\min\left\{\left(\frac{30 \cdot 500}{1200}\right);\ 3,15 \cdot \left[1 - e^{-\frac{0,036}{0,0791}}\right] \cdot 6,80 \cdot \left(\frac{1,70}{0,625}\right)^{1/2}\right\} = \min\{12,5;\ 12,9\} = 12,5\ MW.$$

Factor Ω: $\Omega = A_f q_{f,d}/\sqrt{A_v A_t} = (30) \cdot (500)/\sqrt{(6,80) \cdot (112)} \approx 543,5\ MJ/m^2$.

Temperature of the fire compartment (eq. 4-11):

$$T_f = 6000 \cdot (1 - e^{-0,1/O}) \cdot \sqrt{O} \cdot (1 - e^{-0,00286\Omega}) + T_0$$

$$T_f = 6000 \cdot (1 - e^{-0,1/(0,0792)}) \cdot \sqrt{0,0791} \cdot (1 - e^{-(0,00286 \cdot 543,5)}) + 293°K$$

$$T_f = 6000 \cdot 0,7171 \cdot 0,2812 \cdot 0,7887 + 273 \approx 955°K + 293°K = 1248°K$$

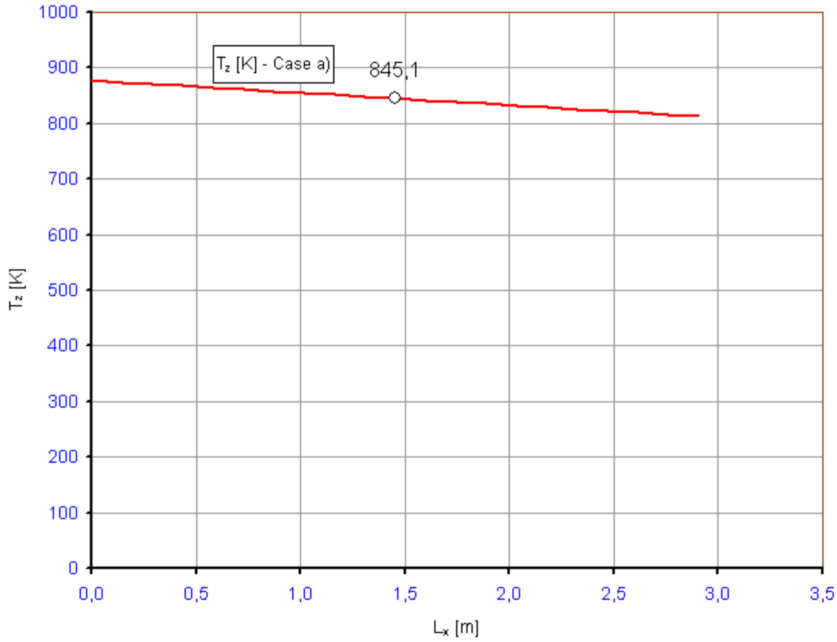
$$T_f = (1248 - 273) = 975°C.$$

Internal gas density, say $\rho = 0,50\ kg/m^3$. Flame height (eq. 4-12):

$$L_L = \max\left\{0;\ h_{eq} \cdot \left[2,37 \cdot \left(\frac{Q}{A_v \rho_g \sqrt{h_{eq} g}}\right)^{2/3} - 1\right]\right\}$$

$$L_L = \max\left\{0;\ (1,70) \cdot \left[2,37 \cdot \left(\frac{12,5}{(6,80)(0,50)\sqrt{(1,70)(9,81)}}\right)^{2/3} - 1\right]\right\} = 2,056\ m.$$

The flame width is the window width: say $w_t = 1,00\ m$. Flame depth: $2h_{eq}/3 = 2 \cdot (1,70)/3 = 1,13\ m$.



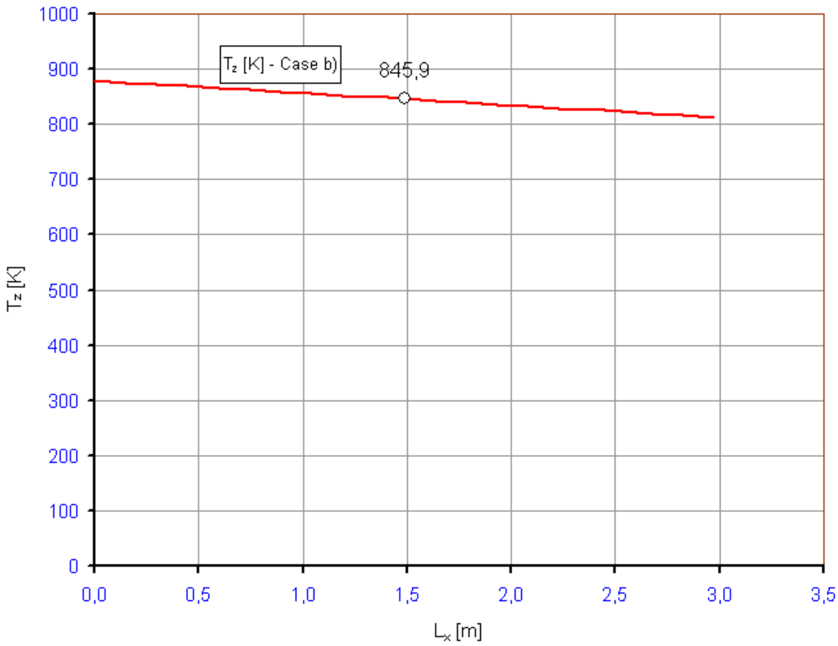


Figure 4.20 Plots eq. (B.15).

The flame temperature along the axis is given by (eq. 4-14):

$$T_z = (T_w - T_0) \cdot \left\{1 - \left[0,4725 \cdot \left(\frac{L_x \cdot w_t}{Q}\right)\right]\right\} + T_0 .$$

For **case a**: $T_z = (877 - 293) \cdot \left\{1 - \left[0,4725 \cdot \left(\frac{L_x \cdot 1,00}{12,5}\right)\right]\right\} + 293 .$

For (say) $L_x = 1,45\ \mathrm{m}$, we get (**case a**):

$$T_z = (877-293)\cdot\left\{1-\left[0,4725\cdot\left(\frac{1,45\cdot 1,00}{12,5}\right)\right]\right\}+293 = 845°K = (845-273) = 572°C.$$

For **case b**: $T_z = (877-293)\cdot\left\{1-\left[0,4725\cdot\left(\frac{L_x\cdot 1,00}{12,5}\right)\right]\right\}+293$.

For (say) $L_x = 1,49\ \mathrm{m}$, we get (**case b**):

$$T_z = (879-293)\cdot\left\{1-\left[0,4725\cdot\left(\frac{1,49\cdot 1,00}{12,5}\right)\right]\right\}+293 = 846°K = (846-273) = 573°C.$$

Flame thickness (say): $d_f = 1,00\ \mathrm{m}$, geometrical characteristic of an external structural element (diameter or side): $d_{eq} = 0,70\ \mathrm{m}$.

Emissivity of flames (eq. 4-15): $\varepsilon_f = 1-e^{-0,3d_f} = 1-e^{-0,3(1,00)} = 0,26$.

Convective heat transfer coefficient (eq. 4-16):

$$\alpha_c = 4,67\cdot(1/d_{eq})^{0,4}\cdot(Q/A_v)^{0,6} = 4,67\cdot(1/0,70)^{0,4}\cdot(12,5/6,80)^{0,6} = 7,8\ \mathrm{W/m^2K}.$$

Forced draught.

Rate of burning or rate of heat release (eq. 4-17):

$$Q = \left(\frac{A_f\cdot q_{f,d}}{\tau_F}\right) = \left(\frac{30,00\cdot 500}{1200}\right) = 12,50\ \mathrm{MW}.$$

Temperature of the fire compartment (eq. 4-18):

$$T_f = 1200\cdot(1-e^{-0,00288\Omega})+T_0 = 1200\cdot(1-e^{-0,00288(543,5)})+293 = 1242°K$$

$$T_f = (1242-273) = 969°C.$$

Flame height (eq. 4-19), with wind speed equal to (say) $u = 6,00\ \mathrm{m/s}$:

$$L_L = \left(1,366\cdot(1/u)^{0,43}\cdot\frac{Q}{\sqrt{A_v}}\right)-h_{eq} = \left(1,366\cdot(1/6,00)^{0,43}\cdot\frac{12,5}{\sqrt{6,80}}\right)-1,70 = 1,33\ \mathrm{m}.$$

The horizontal projection of flames is given by (eq. 4-20):

$$L_H = 0,605\cdot(u^2/h_{eq})^{0,22}\cdot(L_L+h_{eq}) = 0,605\cdot(6,00^2/1,70)^{0,22}\cdot(1,33+1,70) = 3,59\ \mathrm{m}.$$

The flame width is given by: $w_f = w_t+0,4L_H = 1,00+0,4\cdot(3,59) = 2,44\ \mathrm{m}$.

The flame length along axis is given by: $L_f = \sqrt{L_L^2+L_H^2} = \sqrt{(1,33)^2+(3,59)^2} = 3,83\ \mathrm{m}$.

The flame temperature at the window is given by (eq. 4-21):

$$T_w = \frac{520}{1-\left[0,3325\cdot\left(\frac{L_f\cdot\sqrt{A_v}}{Q}\right)\right]}+T_0 = \frac{520}{1-\left[0,3325\cdot\left(\frac{3,83\cdot\sqrt{6,80}}{12,5}\right)\right]}+293 = 1001°K$$

$T_w = (1001-273) = 728°C$, with $L_f\sqrt{A_v}/Q = (3,83)\sqrt{6,80}/12,5 = 0,8<1$ (case applicable). The emissivity of flames at the window may be taken as $\varepsilon_f = 1,00$. The flame temperature along the axis is given by (eq. 4-22):

$$T_z = (T_w-T_0)\cdot\left\{1-\left[0,3325\cdot\left(\frac{L_x\cdot\sqrt{A_v}}{Q}\right)\right]\right\}+T_0$$

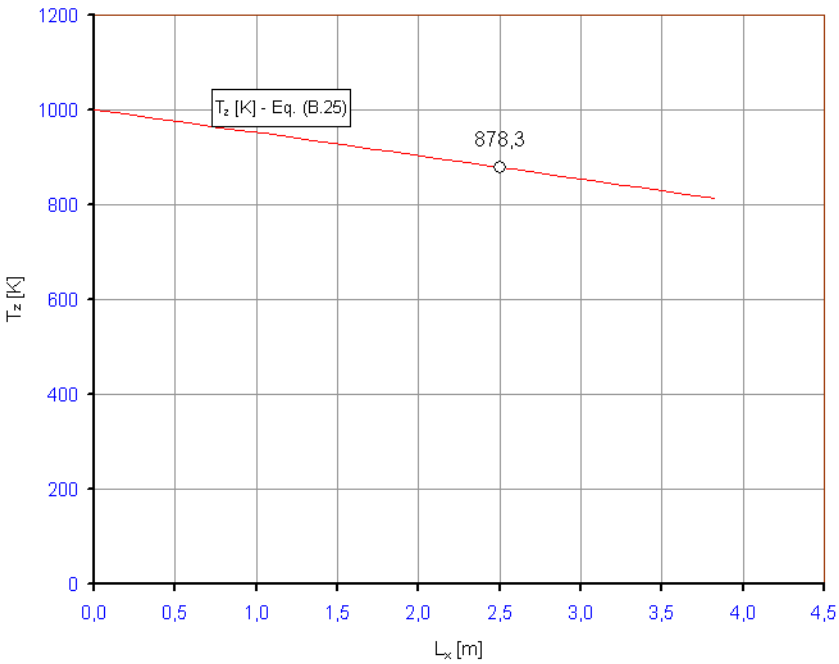


Figure 4.21 Plots eq. (B.25).

$$T_z = (1001 - 293) \cdot \left\{1 - \left[0,3325 \cdot \left(\frac{2,50 \cdot \sqrt{6,80}}{12,5}\right)\right]\right\} + 293 = 878°K = (878 - 273) = 605°C,$$

with (say) $L_x = 2,50$ m. The convective heat transfer coefficient is given by (eq. 4-24):

$$\alpha_c = 9,8 \cdot (1/d_{eq})^{0,4} \cdot \left(\frac{Q}{17,5A_v} + \frac{u}{1,6}\right)^{0,6}$$

$$\alpha_c = 9,8 \cdot (1/0,70)^{0,4} \cdot \left(\frac{12,5}{17,5 \cdot (6,80)} + \frac{6,00}{1,6}\right)^{0,6} = 25,4 \text{ W/m}^2\text{K}.$$

example-end

EXAMPLE 4-X- Sec. B.4.1, Characteristic of fire and flames: with awning or balcony - **test3**

Given: Consider the same assumptions in the example above. Find the flame height L_L and the horizontal projection L_H of the flame if an awning or balcony (with horizontal projection: $W_a = 0,50$ m) is located at the level of the top of the window on its whole width.

[*Reference sheet:* Annex B]-[*Cell-Range: A267:O267-A340:O340*].

Solution: **Case a)**, wall above and $h_{eq} \le 1,25w_t$.

The flame height L_L given in eq. 4-12 is decreased by $W_a \cdot (1 + \sqrt{2})$:

$$L_L^* = L_L - W_a \cdot (1 + \sqrt{2}) = 2,06 - 0,50 \cdot (1 + \sqrt{2}) = 0,85 \text{ m}.$$

The horizontal projection of the flame L_H given in (6), is increased by W_a:

$$\begin{cases} L_H^* = L_H + W_a = 0,57 + 0,50 = 1,07 \text{ m} \\ L_H^* = L_H + W_a = 0,68 + 0,50 = 1,18 \text{ m} \\ L_H^* = L_H + W_a = 0,71 + 0,50 = 1,21 \text{ m} \end{cases}$$

Case b), no wall above or $h_{eq} > 1,25 w_t$.

The flame height L_L given in (3) is decreased by W_a:

$L_L^* = L_L - W_a = 2,06 - 0,50 = 1,56$ m.

The horizontal projection of the flame L_H given in (6), with the above mentioned value of L_L^*, is increased by W_a:

$$\begin{cases} L_H = 0,6 h_{eq} L_L^* / h_{eq}{}^{1/3} \\ L_L^* = 1,56 \text{ m} \end{cases}$$

$L_H = 0,6 h_{eq} (L_L^* / h_{eq})^{1/3} = 0,6 \cdot (1,70)(1,56/1,70)^{1/3} = 0,99$ m.

$L_H^* = W_a + L_H = 0,50 + 0,99 = 1,49$ m.

example-end

4.3 References [Section 4]

BS EN 1991-1-2. *Eurocode 1: Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire.* 26 November 2002.

EN 1991-1-2:2002/AC:2013. Eurocode 1: Actions on structures - Part 1-2: General actions - Actions on structures exposed to fire. CEN Brussels, February 2013.

EN 1991-1-2 (2002) (English): *Eurocode 1: Actions on structures - Part 1-2: General actions - Actions on structures exposed to fire* [Authority: The European Union Per Regulation 305/2011, Directive 98/34/EC, Directive 2004/18/EC]. European Committee for Standardisation.

Section 28 EN 1991-1-4 Annex B

28.1 Procedure 1 for determining the structural factor $c_s c_d$

The structural factor $c_s c_d$ should take into account the effect on wind actions from the non simultaneous occurrence of peak wind pressures on the surface (c_s) together with the effect of the vibrations of the structure due to turbulence (c_d). The detailed procedure for calculating the structural factor $c_s c_d$ is given in expression below (Eq. 6.1 – EN 1991-1-4 Section 6.3 "*Detailed procedure*"). This procedure can only be used if particular conditions given in 6.3.1 (2) apply.

$$c_s c_d = \frac{1 + 2k_p \cdot I_v(z_s) \cdot \sqrt{B^2 + R^2}}{1 + 7 \cdot I_v(z_s)} \qquad \text{(Eq. 28-1)}$$

where:

- z_s is the reference height for determining the structural factor, see Figure 6.1.[1]
- k_p is the peak factor defined as the ratio of the maximum value of the fluctuating part of the response to its standard deviation
- I_v is the turbulence intensity defined in 4.4 (EN 1991-1-4)
- B^2 is the background factor, allowing for the lack of full correlation of the pressure on the structure surface
- R^2 is the resonance response factor, allowing for turbulence in resonance with the vibration mode.

The size factor c_s takes into account the reduction effect on the wind action due to the non simultaneity of occurrence of the peak wind pressures on the surface and may be obtained from Expression:

$$c_s = \frac{1 + 7 \cdot I_v(z_s)\sqrt{B^2}}{1 + 7 \cdot I_v(z_s)} \qquad \text{(Eq. 28-2)}$$

(1) For structures where Figure 6.1 does not apply z_s may be set equal to h, the height of the structure.

The dynamic factor c_d takes into account the increasing effect from vibrations due to turbulence in resonance with the structure and may be obtained from Expression:

$$c_d = \frac{1 + 2k_p \cdot I_v(z_s)\sqrt{B^2 + R^2}}{1 + 7 \cdot I_v(z_s) \cdot \sqrt{B^2}}. \qquad \textbf{(Eq. 28-3)}$$

The procedure to be used to determine k_p, B and R may be given in the National Annex. A recommended procedure is given in Annex B. An alternative procedure is given in Annex C. As an indication to the users the differences in $c_s c_d$ using Annex C compared to Annex B does not exceed approximately 5%.

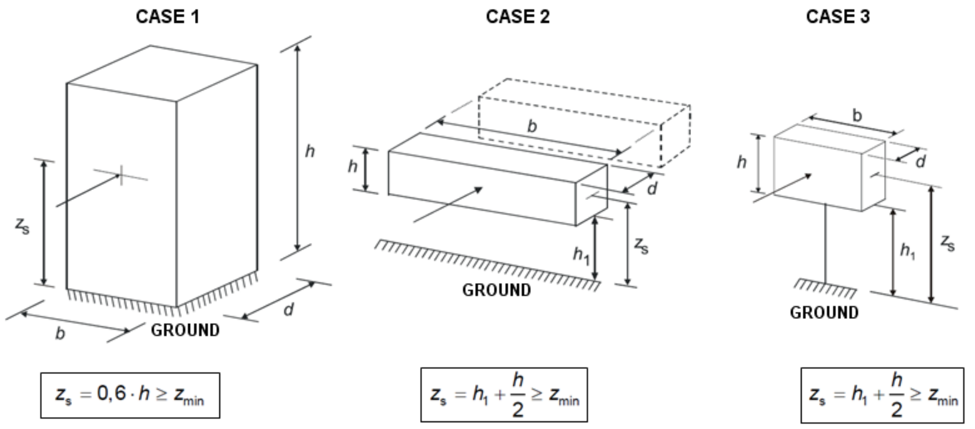


Figure 28.1 From Figure 6.1 - General shapes of structures covered by the design procedure.

WIND TURBULENCE. For heights $z < 200$ m the turbulent length scale $L(z)$ may be calculated using Expression:

$$\begin{cases} L(z) = L_t \cdot \left(\dfrac{z}{200}\right)^{\alpha} & for \quad z \geq z_{min} \\ L(z) = L_t \cdot \left(\dfrac{z_{min}}{200}\right)^{\alpha} & for \quad z < z_{min} \end{cases} \qquad \textbf{(Eq. 28-4)}$$

with $L_t = 300$ m and z, z_{min} in meters. The exponent is equal to:

$$\alpha = 0,67 + 0,05 \cdot \ln(z_0),$$

where the roughness length z_0 is measured in metres. The wind distribution over frequencies is expressed by the non-dimensional power spectral density function:

$$S_L(z, n) = \frac{n \cdot S_v(z, n)}{\sigma_v^2} = \frac{6,8 \cdot f_L(z, n)}{(1 + 10,2 \cdot f_L(z, n))^{5/3}} \qquad \textbf{(Eq. 28-5)}$$

where $S_v(z, n)$ is the one-sided variance spectrum, and $f_L(z, n) = n \cdot L(z)/v_m(z)$ is a non-dimensional frequency determined by the frequency $n = n_{1,x}$, the natural frequency of the structure in Hz, by the mean velocity $v_m(z)$ and the turbolence length scale $L(z)$.

BACKGROUND FACTOR. It may be calculated using Expression:

$$B^2 = \frac{1}{1 + 0,9 \cdot \left(\frac{b + h}{L(z_s)}\right)^{0,63}} \qquad \textbf{(Eq. 28-6)}$$

where:

- b, h is the width and height of the structure, as given in Figure 6.1
- $L(z_s)$ is the turbulent length scale given in B.1(1) at reference height z_s as defined in Figure 6.1.

The peak factor can be expressed by the following equation:

$$k_p = \sqrt{2 \cdot \ln(\nu T)} + \frac{0,6}{\sqrt{2 \cdot \ln(\nu T)}} \qquad \textbf{(Eq. 28-7)}$$

with the limit $k_p \geq 3$ to which corresponds $\nu_{lim} = 0,08\ Hz$. From the expression above, we have:

- ν the up-crossing frequency
- T the averaging time for the mean wind velocity, $T = 600$ seconds.

The up-crossing frequency should be obtained from expression:

$$\nu = n_{1,x} \cdot \sqrt{\frac{R^2}{B^2 + R^2}} \geq \nu_{lim} = 0,08\ Hz \qquad \textbf{(Eq. 28-8)}$$

where $n_{1,x}$ is the natural frequency of the structure, which may be determined using Annex F.

RESONANCE RESPONSE FACTOR. It should be determined using expression:

$$R^2 = \frac{\pi^2}{2\delta} \cdot S_L(z_s, n_{1,x}) \cdot R_h(\eta_h) \cdot R_b(\eta_b) \qquad \textbf{(Eq. 28-9)}$$

where:

- δ is the total logarithmic decrement of damping given in F.5 (Annex F)
- S_L is the non-dimensional power spectral density function
- R_h, R_b are the aerodynamic admittance functions for a fundamental mode shape:

$$R_h = \frac{1}{\eta_h} - \frac{1}{2\eta_h^2} \cdot [1 - \exp(-2 \cdot \eta_h)] \text{ with } R_h = 1 \text{ for } \eta_h = 0 \qquad \textbf{(Eq. 28-10)}$$

$$R_b = \frac{1}{\eta_b} - \frac{1}{2\eta_b^2} \cdot [1 - \exp(-2 \cdot \eta_b)] \text{ with } R_b = 1 \text{ for } \eta_b = 0$$ **(Eq. 28-11)**

with: $\eta_h = \frac{4,6 \cdot h}{L(z_s)} \cdot f_L(z_s, n_{1,x})$ and $\eta_b = \frac{4,6 \cdot b}{L(z_s)} \cdot f_L(z_s, n_{1,x})$.

28.2 Number of loads for dynamic response

Figure below shows the number of times N_g, that the value ΔS of an effect of the wind is reached or exceeded during a period of 50 years.

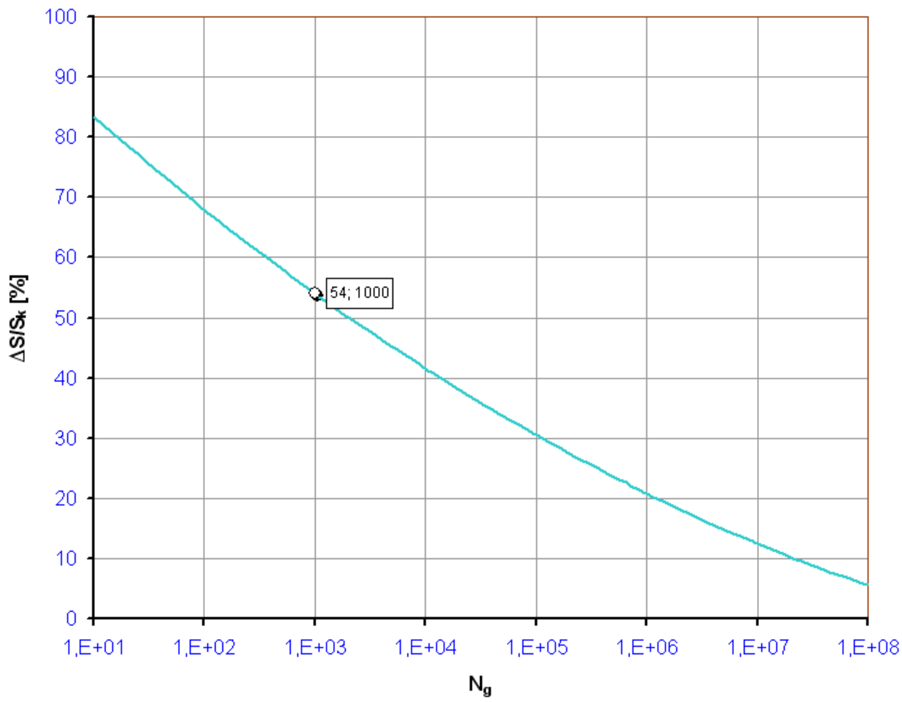


Figure 28.2 Number of gust loads N_g (= 1000) for an effect Δ_S/S_k (= 54%) during a 50 years period.

The relationship between $\Delta S/S_k$ and N_g is given by expression:

$$\frac{\Delta S}{S_k} = 0,7 \cdot (\log(N_g))^2 - 17,4 \cdot \log(N_g) + 100 .$$ **(Eq. 28-12)**

28.3 Service displacement and accelerations for serviceability assessments of a vertical structure

The maximum along-wind displacement is determined from the equivalent static wind force F_w acting on a structure (see EN 1991-1-4, Section 5.3 - "*Wind forces*"). The standard deviation of the characteristic along-wind acceleration of the structural point at height z should be obtained using Expression:

$$\sigma_{a,x}(z) = \frac{c_f \cdot \rho \cdot b \cdot I_v(z_s) \cdot v_m^2(z_s)}{m_{1,x}} \cdot R \cdot K_x \cdot \Phi_{1,x}(z) \qquad \text{(Eq. 28-13)}$$

where:

- c_f is the force coefficient (see Section 7)
- ρ is the air density
- b is the width of the structure as defined in Figure 6.1
- z_s is the reference height as defined in Figure 6.1
- $I_v(z_s)$ is the turbulence intensity at height $z = z_s$ above ground
- $v_m(z_s)$ is the mean wind velocity for $z = z_s$
- R is the square root of resonant response
- K_x is the non-dimensional coefficient
- $m_{1,x}$ is the along wind fundamental equivalent mass (see Annex F, Section F.4(1))
- $n_{1,x}$ is the fundamental frequency of along
- $\Phi_{1,x}(z)$ is the fundamental along wind modal shape.(2)

The non dimensional coefficient, K_x, is defined by:

$$K_x = \frac{\int_0^h v_m^2(z)\Phi_{1,x}(z)dz}{v_m^2(z_s)\int_0^h \Phi_{1,x}(z)dz} \approx \frac{(2\zeta+1)\cdot\left\{(\zeta+1)\cdot\left[\ln\left(\frac{z_s}{z_0}\right)+0,5\right]\cdot -1\right\}}{(\zeta+1)^2\cdot\ln\left(\frac{z_s}{z_0}\right)} \qquad \text{(Eq. 28-14)}$$

where:

- z_0 is the roughness length (see Table 4.1)
- ζ is the exponent of the mode shape (see Annex F).

The characteristic peak accelerations are obtained by multiplying the standard deviation in Eq. 28-13 by the peak factor in Eq. 28-7 using the natural frequency as upcrossing frequency, i.e. $\nu = n_{1,x}$.

28.4 Verification tests

EN1991-1-4_(C).XLS. 6.32 MB. Created: 02 April 2013. Last/Rel.-date: 02 April 2013. Sheets:

— Splash

— Annex B.

(2) As a first approximation the expressions given in Annex F be used.

EXAMPLE 28-A- Procedure 1 for determining the structural factor $c_s c_d$ - **test1**

Given: Find the turbulent length scale $L(z)$ and the power spectral density function $S_L(z, n)$ at an actual height $z_{act} = 30$ m above ground level at the site of the structure. Suppose a displacement height $h_{dis} = 10$ m, a mean wind velocity (mean return period: $N = 100$ years) equal to $v_m(z_{act} - h_{dis}) = 36,4$ m/s and $n_{1,x} = 0,5$ Hz (lower natural frequency of the structure: mode shape 1). Choose terrain category "0".

[*Reference sheet:* Annex B]-[*Cell-Range: A1:N1-A73:N73*].

Solution: Entering Table 4.1 ("*Terrain categories and terrain parameters*") for terrain category "0": $z_0 = 0,003$ m and $z_{min} = 1$ m.

We have:

$$z = z_{act} - h_{dis} = 30 - 10 = 20 \text{ m}, \; \alpha = 0,67 + 0,05 \cdot \ln(z_0) = 0,67 + 0,05 \cdot \ln(0,003) = 0,38$$

For $z = 20 \text{ m} > z_{min} = 1$ m:

$$L(z) = L_t \cdot \left(\frac{z}{200}\right)^{\alpha} = 300 \cdot \left(\frac{20}{200}\right)^{0,38} = 125 \text{ m}.$$

Non-dimensional frequency:

$$f_L(z, n) = \frac{n \cdot L(z)}{v_m(z)} = \frac{0,5 \cdot 125}{36,4} = 1,7 \; [-].$$

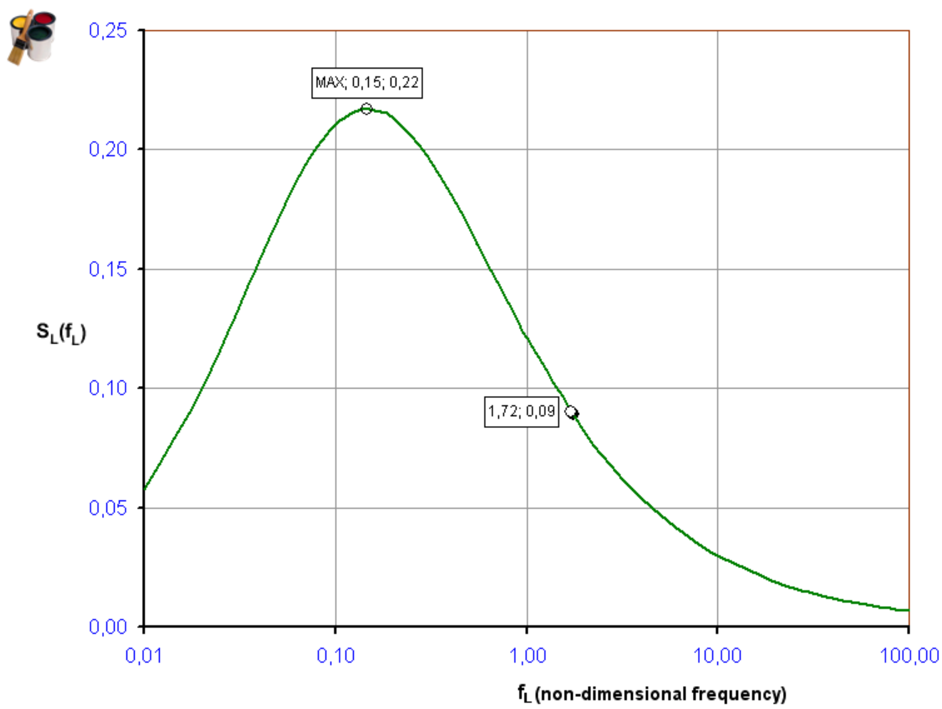


Figure 28.3 Power spectral density function for terrain Category 0 and natural frequency equal to 0,5 Hz.

Power spectral density function (see plot above):

$$S_L(z, n) = \frac{n \cdot S_v(z, n)}{\sigma_v^2} = \frac{6,8 \cdot f_L(z, n)}{(1 + 10,2 \cdot f_L(z, n))^{5/3}} = \frac{6,8 \cdot 1,7}{(1 + 10,2 \cdot 1,7)^{5/3}} = 0,09$$

for $z = z_{act} - h_{dis} = 30 - 10 = 20$ m and $n = n_{1,x} = 0,5$ Hz.

example-end

EXAMPLE 28-B- Procedure 1 for determining the structural factor c_sc_d - **test2**

Given: Find the structural factor c_sc_d as defined in 6.3.1 (see EN 1991-1-4 Section 6) for a building with a central core plus peripheral columns and shear bracings. Assume that the structure has a parallelepiped shape and a total logarithmic decrement of dumping (as given in F.5) equal to $\delta = 5\%$. The width and the height of the structure are equal to $b = 20$ m and $h = 60$ m respectively. Assume a mean wind velocity (mean return period: $N = 50$ years, in terrain category "0") equal to $v_m(z_s) = 37,4$ m/s for $z_s = 0,6h$.

[*Reference sheet:* Annex B]-[*Cell-Range: A77:N77-A224:N224; A338:N338-A360:N360*].

Solution: We have: $z_s = 0,6h = 0,6 \cdot 60 = 36$ m with $v_m(z_s) = 37,4$ m/s.

For $z_s = 36\text{ m} > z_{min} = 1$ m, we get (with $n = n_{1,x} = 0,5$ Hz):

$$L(z) = L_t \cdot \left(\frac{z}{200}\right)^{\alpha} = 300 \cdot \left(\frac{36}{200}\right)^{0,38} = 156,5 \text{ m}.$$

$$f_L(z_s, n) = \frac{n \cdot L(z_s)}{v_m(z_s)} = \frac{0,5 \cdot 156,5}{37,4} = 2,09 \text{ [-]}.$$

$$S_L(z_s, n) = \frac{n \cdot S_v(z_s, n)}{\sigma_v^2} = \frac{6,8 \cdot f_L(z_s, n)}{(1 + 10,2 \cdot f_L(z_s, n))^{5/3}} = \frac{6,8 \cdot 2,09}{(1 + 10,2 \cdot 2,09)^{5/3}} = 0,08.$$

Background factor:

$$B^2 = \frac{1}{1 + 0,9 \cdot \left(\frac{b+h}{L(z_s)}\right)^{0,63}} = \frac{1}{1 + 0,9 \cdot \left(\frac{20+60}{156,5}\right)^{0,63}} = 0,629 \rightarrow B = \sqrt{B^2} = 0,793.$$

Resonance response factor (variables):

$$\eta_h = \frac{4,6 \cdot h}{L(z_s)} \cdot f_L(z_s, n_{1,x}) = \frac{4,6 \cdot 60}{156,5} \cdot 2,09 = 3,69$$

$$\eta_b = \frac{4,6 \cdot b}{L(z_s)} \cdot f_L(z_s, n_{1,x}) = \frac{4,6 \cdot 20}{156,5} \cdot 2,09 = 1,23.$$

Aerodynamic admittance functions (for fundamental mode shape 1):

$$R_h = \frac{1}{\eta_h} - \frac{1}{2\eta_h^2} \cdot [1 - \exp(-2 \cdot \eta_h)] = \frac{1}{3,69} - \frac{1}{2(3,69)^2} \cdot [1 - \exp(-2 \cdot 3,69)] = 0,234$$

$$R_b = \frac{1}{\eta_b} - \frac{1}{2\eta_b^2} \cdot [1 - \exp(-2 \cdot \eta_b)] = \frac{1}{1,23} - \frac{1}{2(1,23)^2} \cdot [1 - \exp(-2 \cdot 1,23)] = 0,511.$$

Resonance response factor allowing for turbulence in resonance with the considered vibration mode of the structure:

$$R^2 = \frac{\pi^2}{2\delta} \cdot S_L(z_s, n_{1,x}) \cdot R_h(\eta_h) \cdot R_b(\eta_b) = \frac{\pi^2}{2 \cdot 0,05} \cdot 0,08 \cdot 0,234 \cdot 0,511 = 0,944 .$$

Up-crossing frequency:

$$\nu = n_{1,x} \cdot \sqrt{\frac{R^2}{B^2 + R^2}} = 0,5 \cdot \sqrt{\frac{0,944}{0,629 + 0,944}} = 0,387 \text{ Hz} > 0,08 \text{ Hz} .$$

Peak factor (with T = 600 s):

$$\nu T = 0,387 \cdot 600 = 232,2 .$$

$$k_p = \sqrt{2 \cdot \ln(\nu T)} + \frac{0,6}{\sqrt{2 \cdot \ln(\nu T)}} = \sqrt{2 \cdot \ln(0,387 \cdot 600)} + \frac{0,6}{\sqrt{2 \cdot \ln(0,387 \cdot 600)}} = 3,48 > 3 .$$

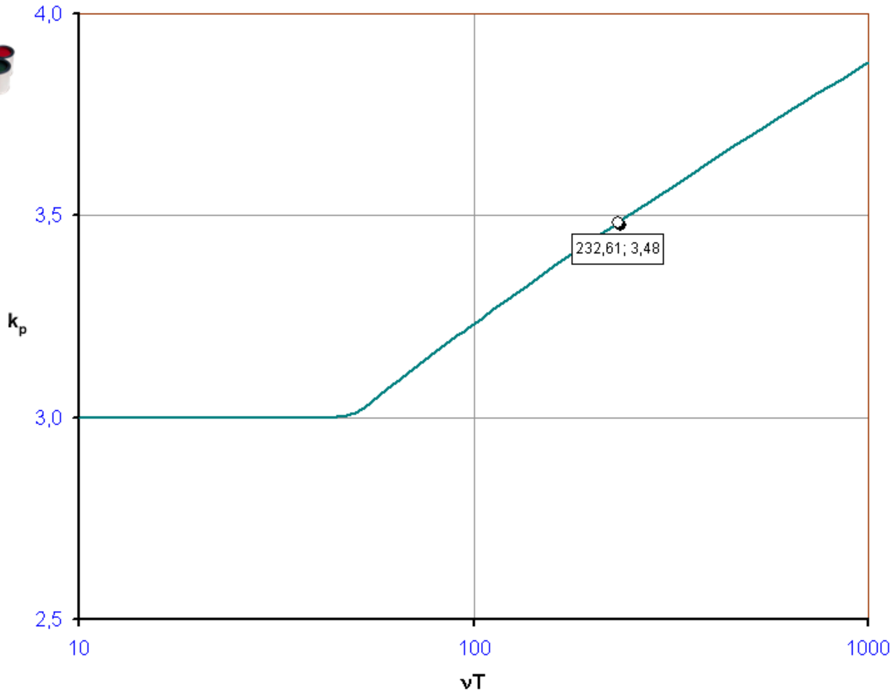


Figure 28.5 Peak factor with natural frequency equal to 0,5 Hz.

From Section 6 - EN 1991-1-4 assuming (say) $I_v(z_s) = 0,10$, we get:

$$c_s = \frac{1 + 7 \cdot I_v(z_s)\sqrt{B^2}}{1 + 7 \cdot I_v(z_s)} = \frac{1 + 7 \cdot 0,10 \cdot 0,793}{1 + 7 \cdot 0,10} = 0,915$$

$$c_d = \frac{1 + 2k_p \cdot I_v(z_s)\sqrt{B^2 + R^2}}{1 + 7 \cdot I_v(z_s) \cdot \sqrt{B^2}} = \frac{1 + 2 \cdot 3,48 \cdot 0,1\sqrt{0,629 + 0,944}}{1 + 7 \cdot 0,1 \cdot 0,793} = 1,204$$

with: $c_s \cdot c_d = 0,915 \cdot 1,204 = 1,102$.

From Eq. 6.1 (see Sec. 6.3.1) we find:

$$c_s c_d = \frac{1 + 2k_p \cdot I_v(z_s) \cdot \sqrt{B^2 + R^2}}{1 + 7 \cdot I_v(z_s)} = \frac{1 + 2 \cdot 3,48 \cdot 0,1 \cdot \sqrt{0,629 + 0,944}}{1 + 7 \cdot 0,1} = 1,102 .$$

An alternative procedure to be used to determine k_p is given in Annex C.

example-end

EXAMPLE 28-C- Number of loads for dynamic response - **test3**

Given: Find the number of times N_g that the value $0,85 \cdot S_k$ of an effect of the wind is reached or exceeded during a period of 50 years, where S_k is the effect of the wind due to a 50 years return period.

[*Reference sheet:* Annex B]-[*Cell-Range: A227:N227-A266:N266*].

Solution: From Eq. B.9, with ΔS expressed as a percentage of the value S_k, we find:

$$\frac{\Delta S}{S_k} = 0,7 \cdot (\log(N_g))^2 - 17,4 \cdot \log(N_g) + 100 = 0,7 \cdot (\log(8))^2 - 17,4 \cdot \log(8) + 100 = 85\%$$

for $N_g = 8$.

example-end

EXAMPLE 28-D- B.4 Service displacement and accelerations for serviceability assessments of a vertical structure - test4

Given: Find the characteristic peak acceleration of the structural point at height $z = h = 60$ m where h is the height of the building (see same assumptions from previous examples). Assume a force coefficient (see Section 7.6 - Eq. 7.9) equal to $c_f = 1,3$ and an air density $\rho = 1,226\ kg/m^3$. The along wind fundamental equivalent mass (see Annex F, Sec. F.4(1)) was calculated previously equal to $m_{1,x} = 10^4\ kg/m$.

[*Reference sheet:* Annex B]-[*Cell-Range: A270:N270-A333:N333*].

Solution: From Annex F, Sec. F.3 ("building with a central core plus peripheral columns or larger columns plus shear bracings"): we find an exponent of the modal shape $\zeta = 1$. Therefore:

$$\Phi_{1,x} = \left(\frac{z}{h}\right)^{\zeta} = \left(\frac{60}{60}\right)^{1} = 1 .$$

Non-dimensional coefficient (see Eq. B.11):

$$K_x \approx \frac{(2\zeta + 1) \cdot \left\{(\zeta + 1) \cdot \left[\ln\left(\frac{z_s}{z_0}\right) + 0,5\right] \cdot -1\right\}}{(\zeta + 1)^2 \cdot \ln\left(\frac{z_s}{z_0}\right)}$$

$$K_x \approx \frac{(2 \cdot 1 + 1) \cdot \left\{(1+1) \cdot \left[\ln\left(\frac{36}{0,003}\right) + 0,5\right] \cdot -1\right\}}{(1+1)^2 \cdot \ln\left(\frac{36}{0,003}\right)} = \frac{56,36}{37,57} = 1,50\,.$$

Square root of resonant response (see calculations previous examples):

$R = \sqrt{R^2} = \sqrt{0,944} = 0,972\,.$

Standard deviation $\sigma_{a,x}(z)$ of the characteristic along-wind acceleration of the structural point at height $z = h = 60\ m$:

$$\sigma_{a,x}(z) = \frac{1,3 \cdot 1,226 \cdot 20 \cdot 0,1 \cdot (37,4)^2}{10^4} \cdot 0,972 \cdot 1,50 \cdot 1,0 = 0,65\,.$$

Using the natural frequency $n_{1,x}$ as up-crossing frequency, we get the new the peak factor:

$$k_{p,n} = \sqrt{2 \cdot \ln(n_{1,x}T)} + \frac{0,6}{\sqrt{2 \cdot \ln(n_{1,x}T)}} = \sqrt{2 \cdot \ln(0,5 \cdot 600)} + \frac{0,6}{\sqrt{2 \cdot \ln(0,5 \cdot 600)}} = 3,56 > 3$$

The characteristic peak acceleration is obtained by multiplying the standard deviation in (B.10) by the peak factor in B.2(3) using the natural frequency $n_{1,x}$ as upcrossing frequency ν:

$k_{p,n} \cdot \sigma_{a,x}(z) = 3,56 \cdot 0,65 = 2,31\ m/s^2\,.$

example-end

28.5 References [Section 28]

EN 1991-1-4:2005/A1:2010. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, April 2010.

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Section 29 **EN 1991-1-4 Annex C**

29.1 Procedure 2 for determining the structural factor $c_s c_d$

The structural factor $c_s c_d$ should take into account the effect on wind actions from the non simultaneous occurrence of peak wind pressures on the surface (c_s) together with the effect of the vibrations of the structure due to turbulence (c_d). The detailed procedure for calculating the structural factor $c_s c_d$ is given in Equation 6.1 (EN 1991-1-4 Section 6.3 “*Detailed procedure*”).

The procedure to be used to determine k_p, B and R may be given in the National Annex. A recommended procedure is given in Annex B. An alternative procedure is given in Annex C. As an indication to the users the differences in $c_s c_d$ using Annex C compared to Annex B does not exceed approximately 5%.

BACKGROUND FACTOR. It may be calculated using Expression:

$$B^2 = \frac{1}{1+\frac{3}{2}\cdot\sqrt{\left(\frac{b}{L(z_s)}\right)^2+\left(\frac{h}{L(z_s)}\right)^2+\left(\frac{b}{L(z_s)}\cdot\frac{h}{L(z_s)}\right)^2}} \qquad \text{(Eq. 29-1)}$$

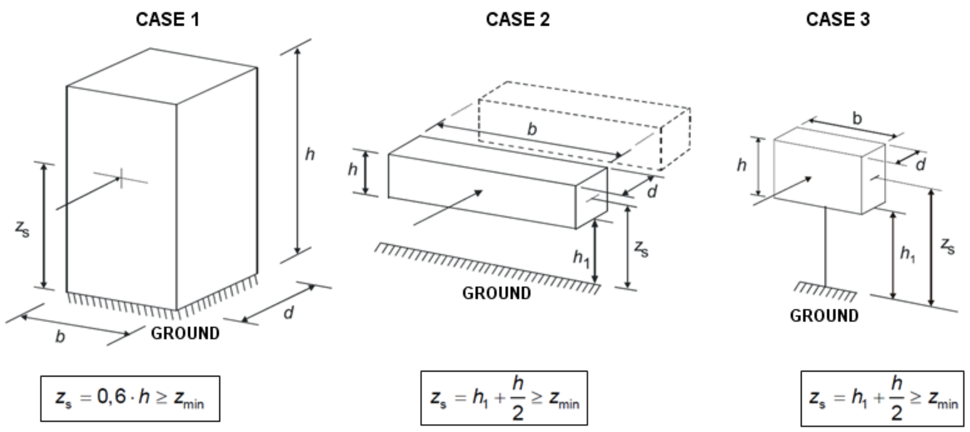


Figure 29.1 From Figure 6.1 - General shapes of structures covered by the design procedure.

where:

- b, h are the width and height of the structure respectively (see Figure 6.1)
- $L(z_s)$ is the turbulent length scale given in B.1 (1) at reference height z_s defined in Figure 6.1

The resonance response factor should be determined using Expression:

$$R^2 = \frac{\pi^2}{2\delta} \cdot S_L(z_s, n_{1,x}) \cdot K_s(n_{1,x}) \qquad \textbf{(Eq. 29-2)}$$

where:

- δ is the total logarithmic decrement of damping given in Annex F
- S_L is the wind power spectral density function given in B.1(2)
- $n_{1,x}$ is the natural frequency of the structure, which may be determined using Annex F
- K_s is the size reduction function given in above:

$$K_s(n_{1,x}) = \frac{1}{1 + \sqrt{(G_y \cdot \phi_y)^2 + (G_z \cdot \phi_z)^2 + \left(\frac{2}{\pi} \cdot G_y \cdot \phi_y \cdot G_z \cdot \phi_z\right)^2}} \qquad \textbf{(Eq. 29-3)}$$

with:

$$\phi_y = \frac{c_y \cdot b \cdot n_{1,x}}{v_m(z_s)}, \quad \phi_z = \frac{c_z \cdot h \cdot n_{1,x}}{v_m(z_s)}$$

where c_y, c_z are the decay constants both equal to 11,5 and $v_m(z_s)$ is the mean wind velocity at reference height z_s (as defind in Figure 6.1).

The constants G introduced in equation above depend on the mode shape variation along the horizontal y-axis and vertical z-axis respectively and should be chosen as follow:

— for buildings with a uniform horizontal mode shape variation and linear vertical mode shape variation $\Phi(y, z) = z/h$ with $K_y = 1$, $K_z = 3/2$: $G_y = 1/2$, $G_z = 3/8$

— for chimneys with a uniform horizontal mode shape variation and parabolic vertical mode shape variation $\Phi(y, z) = z^2/h^2$ with $K_y = 1$, $K_z = 5/3$: $G_y = 1/2$, $G_z = 5/18$

— for bridges with a sinusoidal horizontal mode shape variation $\Phi(y, z) = \sin(\pi \cdot y/b)$ with $K_y = 4/\pi$, $K_z = 1$: $G_y = 4/\pi^2$, $G_z = 1/2$.

29.2 Number of loads for dynamic response

See Annex B, Section B.3.

29.3 Service displacement and accelerations for serviceability assessments

The maximum along-wind displacement is the static displacement determined from the equivalent static wind force defined in 5.3. The standard deviation $\sigma_{a,x}$ of the characteristic along-wind acceleration of the structural point with coordinates (y,z) is approximately given by Expression:

$$\sigma_{a,x} = c_f \cdot \rho \cdot I_v(z_s) \cdot v_m^2(z_s) \cdot \frac{R \cdot K_y \cdot K_z \cdot \Phi(y,z)}{\mu_{ref} \cdot \Phi_{max}} \qquad \text{(Eq. 29-4)}$$

where:

- c_f is the force coefficient (see Section 7.6, Eq. 7.9)
- ρ is the air density (see Section 4.5)
- z_s is the reference height, see Figure 6.1
- $I_v(z_s)$ is the turbulence intensity at height z_s above ground, see 4.4(1)
- $v_m(z_s)$ is the characteristic mean wind velocity at height z_s, see 4.3.1(1)
- R is the square root of the resonant response, see C.2(4)
- K_y, K_z are the constants given in C.2(6)
- μ_{ref} is the reference mass per unit area (see Annex F, SectionF.5(3))
- $\Phi(y, z)$ is the mode shape
- Φ_{max} is the mode shape at the point with maximum amplitude

The characteristic peak accelerations are obtained by multiplying the standard deviation $\sigma_{a,x}$ by the peak factor k_p (see Annex B, Section in B.2(3)) using the natural frequency as upcrossing frequency, i.e. $\nu = n_{1,x}$.

29.4 Verification tests

EN1991-1-4_(D).XLS. 6.36 MB. Created: 11 April 2013. Last/Rel.-date: 11 April 2013. Sheets:

— Splash

— Annex C.

EXAMPLE 29-A- Procedure 2 for determining the structural factor $c_s c_d$ - **test1**

Given: Assume a multi spam (simply-supported) bridge carrying two line of traffic. The construction consists of a reinforced concrete slab supported by steel girders with welded cover plate. The longest spam length is equal to $b = 40$ m and the bridge width is equal to $d = 13$ m (see Figure 6.1). The reference mass per unit area of the bridge is $\mu_{ref} = 2500\ \mathrm{kg/m^2}$ according to Annex F, Sec. F.5(3). The height of the piles of the bridge is $h_1 = 41,5$ meters and the entire height of the bridge (as defined in Figure 6.1) is assumed to be $h = 3$ meters (deck, security barrier and the vehicles during bridge service life with traffic). Assuming: $c_f = c_{fx,0} = 1,3$ (force coefficient), $z_s = h_1 + 0,5h = 43$ m (reference height, as defined in Figure 6.1), $v_m(z_s) = 37,4$ m/s (mean wind velocity at height z_s above ground), terrain category "0", find:

1) the characteristic peak acceleration for a natural frequency $n_{1,x} = 1,5$ Hz of the bridge (1^{th} mode shape)

2) the structural factor $c_s c_d$ for the deck of the bridge.

[*Reference sheet:* Annex B]-[*Cell-Range: A77:N77-A360:N360*].

Solution: **1)** Entering Table 4.1 ("*Terrain categories and terrain parameters*"): $z_0 = 0,003$ m, $z_{min} = 1$ m.

For $z = z_s = 43 \text{ m} \geq z_{min}$:

$$L(z_s) = 300 \cdot \left(\frac{43}{200}\right)^{0,38} = 167,4 \text{ m, with}$$

$$\alpha = 0,67 + 0,05 \cdot \ln(z_0) = 0,67 + 0,05 \cdot \ln(0,003) = 0,380.$$

Non-dimensional frequency:

$$f_L(z_s, n_{1,x}) = \frac{n_{1,x} \cdot L(z_s)}{v_m(z_s)} = \frac{1,5 \cdot 167,4}{37,4} = 6,71 \ [-]$$

Power spectral density function:

$$S_L(z_s, n) = \frac{n \cdot S_v(z_s, n)}{\sigma_v^2} = \frac{6,8 \cdot f_L(z_s, n)}{(1 + 10,2 \cdot f_L(z_s, n))^{5/3}} = \frac{6,8 \cdot 6,71}{(1 + 10,2 \cdot 6,71)^{5/3}} = 0,039 \approx 0,04.$$

Background factor:

$$B^2 = \frac{1}{1 + \frac{3}{2} \cdot \sqrt{\left(\frac{b}{L(z_s)}\right)^2 + \left(\frac{h}{L(z_s)}\right)^2 + \left(\frac{b}{L(z_s)} \cdot \frac{h}{L(z_s)}\right)^2}}$$

$$B^2 = \frac{1}{1 + \frac{3}{2} \cdot \sqrt{\left(\frac{40}{167,4}\right)^2 + \left(\frac{3}{167,4}\right)^2 + \left(\frac{40}{167,4} \cdot \frac{3}{167,4}\right)^2}} = 0,736 \rightarrow B = \sqrt{B^2} = 0,858.$$

Size reduction factor (variables), with $n_{1,x} = 1,5$ Hz:

$$\phi_y = \frac{c_y \cdot b \cdot n_{1,x}}{v_m(z_s)} = \frac{11,5 \cdot 40 \cdot 1,5}{37,4} = 18,45, \ \phi_z = \frac{c_z \cdot h \cdot n_{1,x}}{v_m(z_s)} = \frac{11,5 \cdot 3 \cdot 1,5}{37,4} = 1,38.$$

Type of structure: bridge with a sinusoidal horizontal mode shape variation with $y/b = 0,5 \rightarrow \Phi(y,z) = \Phi_{max} = 1$. Therefore, from Table C.1: $G_y = 4/\pi^2 = 0,405$, $G_z = 0,5$, $K_y = 4/\pi = 1,273$, $K_z = 1$.

Size reduction function:

$$K_s(n_{1,x}) = \frac{1}{1 + \sqrt{(G_y \cdot \phi_y)^2 + (G_z \cdot \phi_z)^2 + \left(\frac{2}{\pi} \cdot G_y \cdot \phi_y \cdot G_z \cdot \phi_z\right)^2}}$$

$$K_s(n_{1,x}) = \frac{1}{1 + \sqrt{(0,405 \cdot 18,45)^2 + (0,5 \cdot 1,38)^2 + \left(\frac{2}{\pi} \cdot 0,405 \cdot 18,45 \cdot 0,5 \cdot 1,38\right)^2}} = 0,109.$$

Resonance response factor (with a total logarithmic decrement of dumping equal to 0,05):

$$R^2 = \frac{\pi^2}{2\delta} \cdot S_L(z_s, n_{1,x}) \cdot K_s(n_{1,x}) = \frac{\pi^2}{2 \cdot 0,05} \cdot 0,039 \cdot 0,109 = 0,420 \rightarrow R = \sqrt{R^2} = 0,648\,.$$

Up-crossing frequency:

$$\nu = n_{1,x} \cdot \sqrt{\frac{R^2}{B^2 + R^2}} = 1,5 \cdot \sqrt{\frac{0,420}{0,736 + 0,420}} = 0,902\ \text{Hz} \geq 0,08\ \text{Hz}\,.$$

Peak factor (with T = 600 s):

$$\nu T = 0,902 \cdot 600 = 541,4\,.$$

$$k_p = \sqrt{2 \cdot \ln(\nu T)} + \frac{0,6}{\sqrt{2 \cdot \ln(\nu T)}} = \sqrt{2 \cdot \ln(541,4)} + \frac{0,6}{\sqrt{2 \cdot \ln(541,4)}} = 3,72 > 3\,.$$

Standard deviation of the characteristic along-wind acceleration of the structural point with coordinates (y; z) = (0,5b; 44,5 m):

$$\sigma_{a,x} = c_f \cdot \rho \cdot I_v(z_s) \cdot v_m^2(z_s) \cdot \frac{R \cdot K_y \cdot K_z \cdot \Phi(y,z)}{\mu_{ref} \cdot \Phi_{max}}\text{, with (say) } I_v(z_s) = 0,10:$$

$$\sigma_{a,x} = 1,3 \cdot 1,226 \cdot 0,1 \cdot (37,4)^2 \cdot \frac{0,648 \cdot 1,273 \cdot 1 \cdot 1}{2500 \cdot 1} = 0,073\,.$$

Using the natural frequency $n_{1,x}$ as up-crossing frequency, we get the new the peak factor:

$$k_{p,n} = \sqrt{2 \cdot \ln(n_{1,x}T)} + \frac{0,6}{\sqrt{2 \cdot \ln(n_{1,x}T)}} = \sqrt{2 \cdot \ln(1,5 \cdot 600)} + \frac{0,6}{\sqrt{2 \cdot \ln(1,5 \cdot 600)}} = 3,85 > 3\,.$$

The characteristic peak acceleration is obtained by multiplying the standard deviation in (B.10) by the peak factor in B.2(3) using the natural frequency $n_{1,x}$ as upcrossing frequency ν:

$$k_{p,n} \cdot \sigma_{a,x}(z) = 3,85 \cdot 0,073 = 0,28\ \text{m/s}^2 \approx 0,3\ \text{m/s}^2\,.$$

2) From Section 6 - EN 1991-1-4 assuming (say) $I_v(z_s) = 0,10$, we get:

$$c_s = \frac{1 + 7 \cdot I_v(z_s)\sqrt{B^2}}{1 + 7 \cdot I_v(z_s)} = \frac{1 + 7 \cdot 0,10 \cdot 0,858}{1 + 7 \cdot 0,10} = 0,94$$

$$c_d = \frac{1 + 2k_p \cdot I_v(z_s)\sqrt{B^2 + R^2}}{1 + 7 \cdot I_v(z_s) \cdot \sqrt{B^2}} = \frac{1 + 2 \cdot 3,72 \cdot 0,1\sqrt{0,736 + 0,420}}{1 + 7 \cdot 0,1 \cdot 0,858} = 1,12$$

with: $c_s \cdot c_d = 0,94 \cdot 1,12 = 1,05$.

From Eq. 6.1 (see Sec. 6.3.1) we find:

$$c_s c_d = \frac{1 + 2k_p \cdot I_v(z_s) \cdot \sqrt{B^2 + R^2}}{1 + 7 \cdot I_v(z_s)} = \frac{1 + 2 \cdot 3,72 \cdot 0,1 \cdot \sqrt{0,736 + 0,420}}{1 + 7 \cdot 0,1} = 1,06\,.$$

An alternative procedure to be used to determine k_p is given in Annex C.

example-end

29.5 References [Section 29]

EN 1991-1-4:2005/A1:2010. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, April 2010.

EN 1991-1-4:2005. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2005 (DAV).

Manual for the design of building structures to Eurocode 1 and Basis of Structural Design April 2010. © 2010 The Institution of Structural Engineers.

Eurocode Load Combinations for Steel Structures. The British Constructional Steelwork Association Limited. BCSA Publication No. 53/10. December 2010.

Section 30 **EN 1991-1-4 Annex E [from Sec. E.1 to Sec. E.1.5.2.5]**

30.1 Vortex shedding

Vortex-shedding occurs when vortices are shed alternately from opposite sides of the structure. This gives rise to a fluctuating load perpendicular to the wind direction. Structural vibrations may occur if the frequency of vortex.shedding is the same as a natural frequency of the structure. This condition occurs when the wind velocity is equal to the critical wind velocity defined in E.1.3.1. Typically, the critical wind velocity is a frequent wind velocity indicating that fatigue, and thereby the number of load cycles, may become relevant. The response induced by vortex shedding is composed of broad-banded response that occurs whether or not the structure is moving, and narrow-banded response originating from motion-induced wind load.

CRITERIA FOR VORTEX SHEDDING. The effect of vortex shedding should be investigated when the ratio of the largest to the smallest crosswind dimension of the structure, both taken in the plane perpendicular to the wind, exceeds 6. The effect of vortex shedding need not be investigated when:

$$v_{crit,\,i} > 1,25 \cdot v_m \quad \textbf{(Eq. 30-1)}$$

where:

- $v_{crit,\,i}$ is the critical wind velocity for mode *i*, as defined in E.1.3.1
- v_m is the characteristic 10 minutes mean wind velocity specified in 4.3.1(1) at the cross section where vortex shedding occurs.

CRITICAL WIND VELOCITY $v_{CRIT,I}$. The critical wind velocity for bending vibration mode *i* is defined as the wind velocity at which the frequency of vortex shedding equals the natural frequency (mode *i*) of the structure or the structural and is given in expression:

$$v_{crit,\,i} = \frac{b \cdot n_{i,\,y}}{St} \quad \textbf{(Eq. 30-2)}$$

where:

- b is the reference width of the cross-section at which resonant vortex shedding occurs and where the modal deflection is maximum for the structure or structural part considered; for circular cylinders the reference width is the outer diameter
- $n_{i,y}$ is the natural frequency of the considered flexural mode *i* of cross-wind vibration; approximations for n1,y are given in F.2
- St is the Strouhal number as defined in E.1.3.2.

The critical wind velocity for ovalling vibration mode *i* of cylindrical shells is defined as the wind velocity at which two times of the frequency of vortex shedding equals a natural frequency of the ovalling mode *i* of the cylindrical shell and is given in expression:

$$v_{crit,i} = \frac{b \cdot n_{i,0}}{2 \cdot St} \qquad \text{(Eq. 30-3)}$$

where:

- b is the outer shell diameter
- St is the Strouhal number as defined in E.1.3.2
- $n_{i,0}$ is the natural frequency of the ovalling mode *i* of the shell.

Strouhal number St (Sec. E.1.3.2). The Strouhal number for different cross-section may be taken from table E.1.

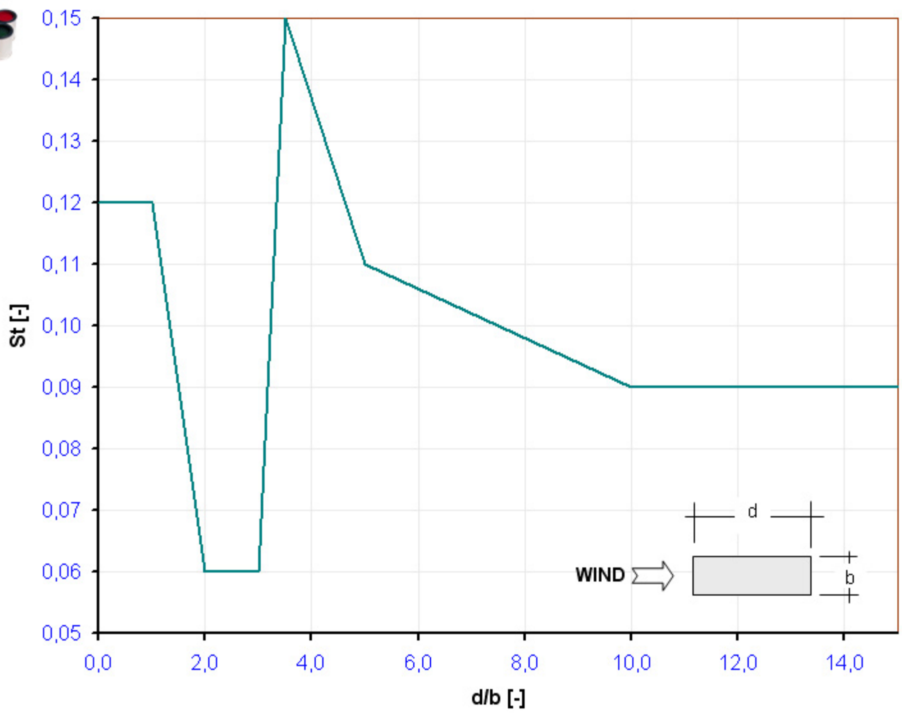
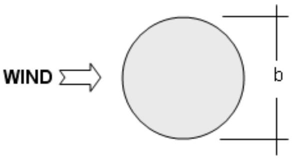
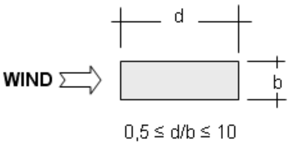
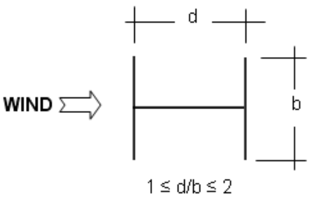
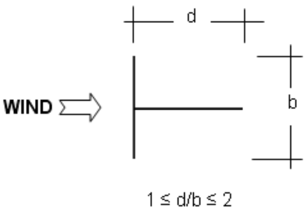
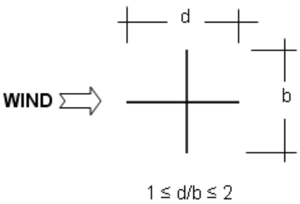
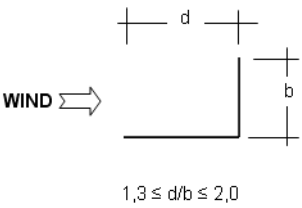


Figure 30.1 From Figure E.1 - Strouhal number (St) for rectangular cross-section with sharp corners.

Cross-section		St
	(for all Re numbers)	0,18
		from Figure E.1
	d/b = 1	0,11
	d/b = 1,5	0,10
	d/b = 2	10,14
	d/b = 1	0,13
	d/b = 2	0,08
	d/b = 1	0,16
	d/b = 2	0,12
	d/b = 1,3	0,11
	d/b = 2,0	0,07

NOTE: extrapolations for Strouhal numbers as function of d/b are not allowed

Table 30.1 From Table E.1 - Strouhal numbers St for different cross-sections.

SCRUTON NUMBER Sc. The susceptibility of vibrations depends on the structural damping and the ratio of structural mass to fluid mass. This is expressed by the Scruton number Sc, which is given in Expression:

$$Sc = \frac{2 \cdot \delta_s \cdot m_{i,e}}{\rho \cdot b^2} \qquad \text{(Eq. 30-4)}$$

where:

- δ_s is the structural damping expressed by the logarithmic decrement
- ρ is the air density under vortex shedding conditions
- $m_{i,e}$ is the equivalent mass m_e per unit length for mode *i* (see Annex F, Sec. F.4(1))
- b is the reference width of the cross-section at which resonant vortex shedding occurs.

REYNOLDS NUMBER Re. The vortex shedding action on a circular cylinder depends on the Reynolds number Re at the critical wind velocity $v_{crit,i}$. The Reynolds number is given in expression:

$$Re(v_{crit,i}) = \frac{b \cdot v_{crit,i}}{\nu} \qquad \text{(Eq. 30-5)}$$

where:

- b is the outer diameter of the circular cylinder
- $\nu = 15 \times 10^{-6}\ m^2/s$ is the kinematic viscosity of the air
- $v_{crit,i}$ is the critical wind velocity (see Sec. E.1.3.1).

30.2 Vortex shedding action

The effect of vibrations induced by vortex shedding should be calculated from the effect of the inertia force per unit length $F_w(s)$, acting perpendicular to the wind direction at location s on the structure and given in expression:

$$F_w(s) = m(s) \cdot (2\pi \cdot n_{i,y})^2 \cdot \Phi_{i,y}(s) \cdot y_{F,max} \qquad \text{(Eq. 30-6)}$$

where:

- $m(s)$ is the vibrating mass of the structure per unit length $[kg/m]$
- $n_{i,y}$ is the natural frequency of the structure
- $\Phi_{i,y}(s)$ is the mode shape of the structure normalised to 1 at the point with the maximum displacement
- $y_{F,max}$ is the maximum displacement over time of the point with $\Phi_{i,y}(s)$ equal to 1 (see Sec. E.1.5).

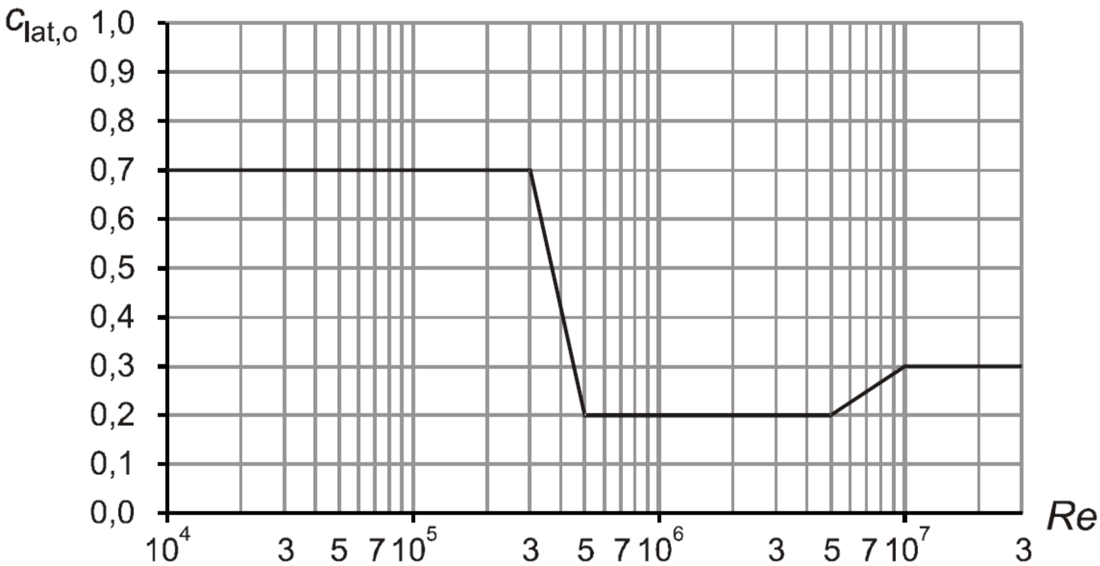


Figure 30.2 From Figure E.2 - Basic value of the lateral force $c_{lat,0}$ versus "Re" for circular cylinders.

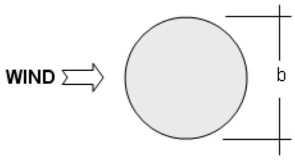
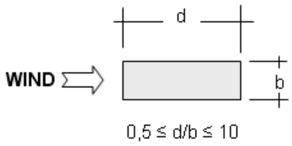
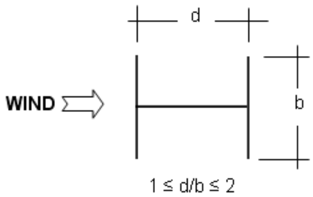
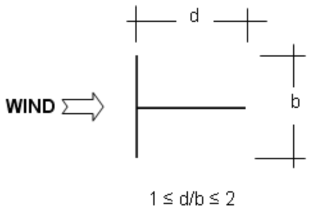
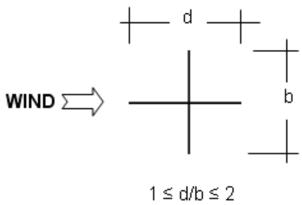
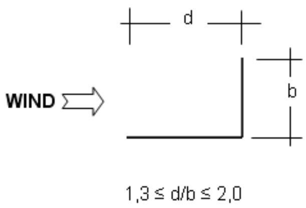
30.3 Calculation of the cross wind amplitude

Two different approaches for calculating the vortex excited cross-wind amplitudes are given in E.1.5.2 and E.1.5.3. The approach given in E.1.5.2 can be used for various kind of structures and mode shapes. It includes turbulence and roughness effects and it may be used for normal climatic conditions. The approach given in E.1.5.3 may be used to calculate the response for vibrations in the first mode of cantilevered structures with a regular distribution of cross wind dimensions along the main axis of the structure. Typically structures covered are chimneys or masts. It cannot be applied for grouped or in-line arrangements and for coupled cylinders. This approach allows for the consideration of different turbulence intensities, which may differ due to meteorological conditions. For regions where it is likely that it may become very cold and stratified flow conditions may occur (e.g. in coastal areas in Northern Europe), approach E.1.5.3 may be used.v

30.3.1 Approach 1 for the calculation of the cross wind amplitudes

CALCULATION OF DISPLACEMENTS. The largest displacement $y_{F,\,max}$ can be calculated using expression:

$$\frac{y_{F,\,max}}{b} = \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_w \cdot c_{lat} \qquad \text{(Eq. 30-7)}$$

Cross-section		$c_{lat,0}$
		From Figure E.2
		1,1
	d/b = 1	0,8
	d/b = 1,5	1,2
	d/b = 2	0,3
	d/b = 1	1,6
	d/b = 2	2,3
	d/b = 1	1,4
	d/b = 2	1,1
	d/b = 1,3	0,8
	d/b = 2,0	1,0

NOTE: extrapolation for lateral force coefficients as function of d/b are not allowed.

Table 30.2 From Table E.2 - Basic value of the lateral force coefficient $c_{lat,0}$ for different cross-sections.

where:

- St is the Strouhal number given in Table E.1
- Sc is the Scruton number given in E.1.3.3
- K_w is the effective correlation length factor (see Sec. E.1.5.2.4)
- K is the mode shape factor (see Sec. E.1.5.2.5)
- c_{lat} is the lateral force coefficient (see Table E.2).

LATERAL FORCE COEFFICIENT c_{LAT}. The basic value $c_{lat,0}$ of the lateral coefficient is given in Table E.2 above. The lateral force coefficient c_{lat} is given in Table E.3 below.

Critical wind velocity ratio	c_{lat}
$\frac{v_{crit,i}}{v_{m,Lj}} \le 0,83$	$c_{lat} = c_{lat,0}$
$0,83 \le \frac{v_{crit,i}}{v_{m,Lj}} \le 1,25$	$c_{lat} = \left(3 - 2,4 \cdot \frac{v_{crit,i}}{v_{m,Lj}}\right) \cdot c_{lat,0}$
$1,25 \le \frac{v_{crit,i}}{v_{m,Lj}}$	$c_{lat} = 0$

where:
$c_{lat,0}$ is the basic value of c_{lat} as given in Table E.2 and, for circular cylinders, in Figure E.2
$v_{crit,i}$ is the critical wind velocity (see Sec. E.1.3.1)
$v_{m,Lj}$ is the mean wind velocity (see 4.3.1) in the centre of the effective correlation length as defined in Figure E.3”.).

Table 30.3 From Table E.3 - Lateral force coefficient c_{lat} versus critical wind velocity ratio, $v_{crit,i}/v_{m,Lj}$.

30.3.2 Correlation length L

The correlation length L_j, should be positioned in the range of antinodes. Examples are given in Figure E.3. For guyed masts and continuous multispan bridges special advice is necessary.

If more than one correlation length is shown, it is safe to use them simultaneously, and the highest value of c_{lat} should be used.

$y_F(s_j)/b$	L_j/b
< 0,1	6
0,1 to 0,6	$4,8 + 12y_F(s_j)/b$
> 0,6	12

Table 30.4 From Table E.4 - Effective correlation length L_j as a function of vibration amplitude $y_F(s_j)$.

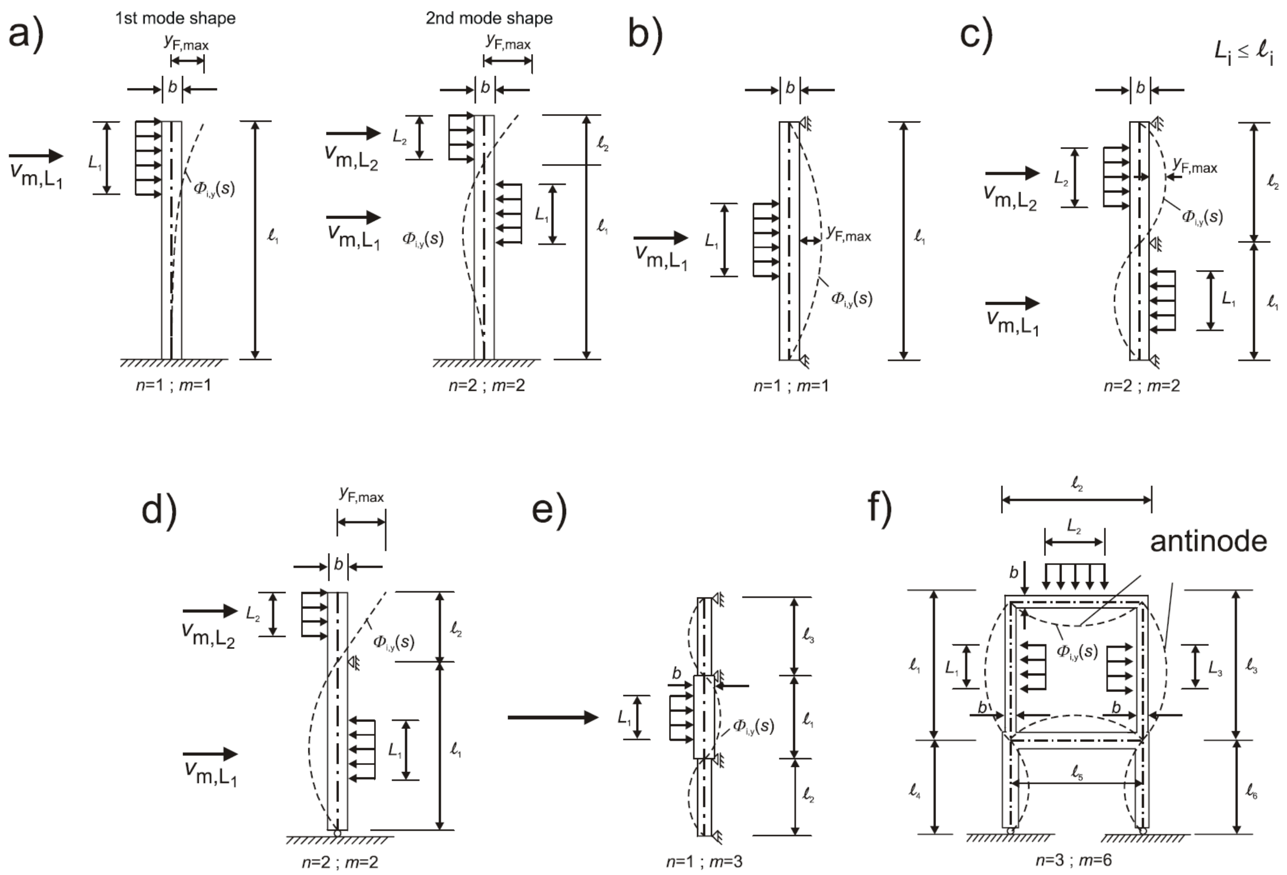


Figure 30.3 From figure E.3 - Examples for application of the correlation length L_j (J = 1, 2, 3).

30.3.3 Effective correlation length factor K_w

The effective correlation length factor K_w is given in expression:

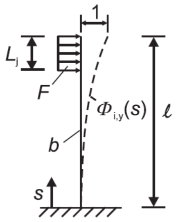
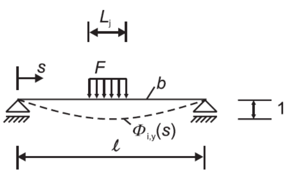
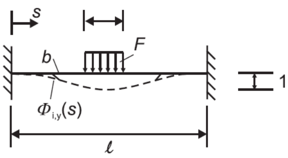
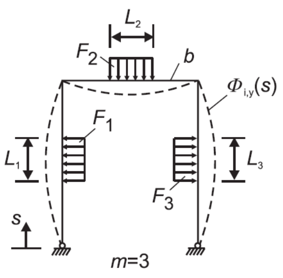
$$K_w = \frac{\sum_{j=1}^{n} \int_{L_j} |\Phi_{i,y}(s)| ds}{\sum_{j=1}^{m} \int_{L_j} |\Phi_{i,y}(s)| ds} \leq 0,6 \qquad \textbf{(Eq. 30-8)}$$

where:

- $\Phi_{i,y}$ is the mode shape (see Annex F, Sec. F.3)
- L_j is the correlation length
- l_j is the length of the structure between two nodes (see Figure E.3); for cantilevered structures it is equal to the height of the structure

- n is the number of regions where vortex excitation occurs at the same time (see Figure E.3)
- m is the number of antinodes of the vibrating structure in the considered mode shape $\Phi_{i,y}$
- s is the coordinate defined in Table E.5.

For some simple structures vibrating in the fundamental cross-wind mode and with the exciting force indicated in Table E.5 the effective correlation length factor K_w can be approximated by the expressions given in Table E.5.

Structure	mode shape $\Phi_{i,y}(s)$	K_w	K
	See F.3 with $\zeta = 2,0$, $n = 1$, $m = 1$	$\frac{3L_j/b}{\lambda}\cdot\left[1-\frac{L_j/b}{\lambda}+\frac{1}{3}\cdot\left(\frac{L_j/b}{\lambda}\right)^2\right]$	0,13
	See Table F.1 with $n = 1$, $m = 1$	$\cos\left[\frac{\pi}{2}\cdot\left(1-\frac{L_j/b}{\lambda}\right)\right]$	0,10
	See Table F.1 with $n = 1$, $m = 1$	$\frac{L_j/b}{\lambda}+\frac{1}{\pi}\cdot\sin\left[\pi\cdot\left(1-\frac{L_j/b}{\lambda}\right)\right]$	0,11
	Modal analysis $n = 3$, $m = 3$	$\left(\sum_{j=1}^{n}\int_{L_j}\lvert\Phi_{i,y}(s)\rvert ds\right)\Big/\left(\sum_{j=1}^{m}\int_{L_j}\lvert\Phi_{i,y}(s)\rvert ds\right)$	0,10

NOTE 1: The mode shape $\Phi_{i,y}(s)$ is taken from F.3. The parameters n and m are defined in Expression (E.8) and in Figure E.3.
NOTE 2: $\lambda = l/b$.

Table 30.5 From Table E.5 - Correlation factor K_w and mode shape factor K for some simple structures.

30.3.4 Mode shape factor

The mode shape factor K is given in expression:

$$K = \frac{\sum_{j=1}^{m} \int_{l_j} |\Phi_{i,y}(s)| ds}{4\pi \cdot \sum_{j=1}^{m} \int_{l_j} \Phi_{i,y}^2(s) ds} \quad \text{(Eq. 30-9)}$$

where:

- m is defined in Sec. E.1.5.2.4(1)
- $\Phi_{i,y}(s)$ is the cross-wind mode shape (see Annex F, Sec. F.3)
- l_j is the length of the structure between two nodes (see Figure E.3).

For some simple structures vibrating in the fundamental cross-wind mode the mode shape factor is given in Table E.5.

30.4 Verification tests

EN1991-1-4_(E).XLS. 6.64 MB. Created: 15 April 2013. Last/Rel.-date: 15 April 2013. Sheets:

— Splash
— Annex E_(a).

EXAMPLE 30-A- Basic parameters for vortex shedding: Strouhal number - **test1**

Given: Find the Strouhal number for:

– a rectangular cross-section with $d/b = 10/3$

– a "H" cross-section with $d/b = 5/4$.

Use data given in Table E.1 and apply the linear interpolation.

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A1:N1-A111:N111*].

Solution: Rectangular cross-section with $d/b \approx 3,33$.

From Figure E.1, linear interpolation between the two points A(3; 0,06) and B(3,5; 0,15):

$$\frac{0,15 - 0,06}{3,5 - 3} = \frac{St - 0,06}{d/b - 3} \approx \frac{St - 0,06}{3,33 - 3} \quad \Rightarrow \quad St = 0,120,$$

see plot below.

"H" cross-section with $d/b = 1,25$.

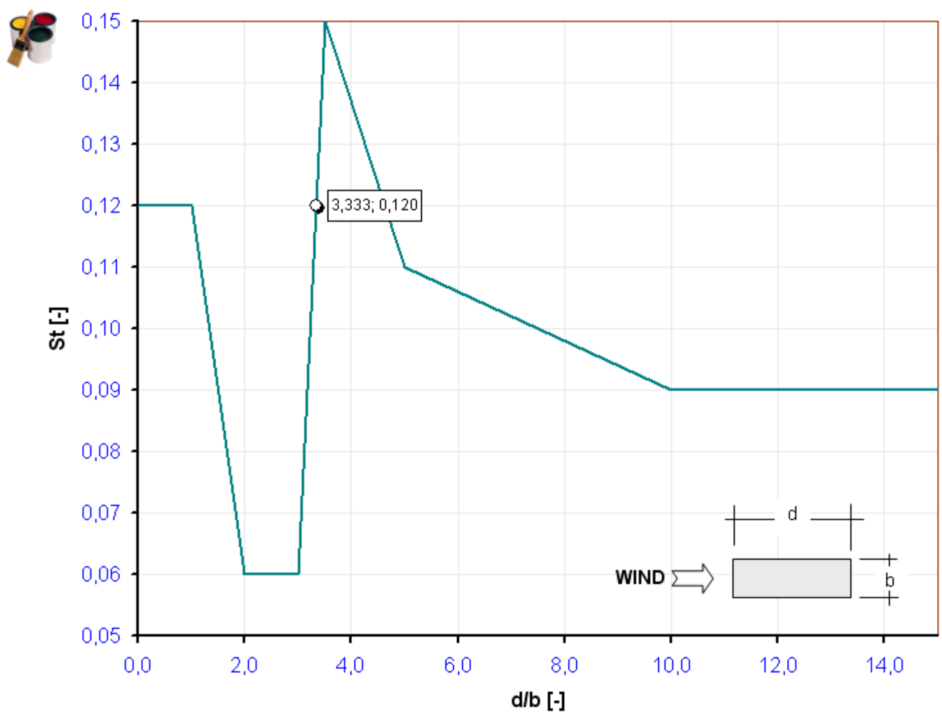


Figure 30.4 From Figure E.1 - Strouhal number (St) for rectangular cross-sections with sharp corners.

Linear interpolation between the two points A(1; 0,11) and B(0,10; 1,5):

$$\frac{0,11 - 0,10}{1,5 - 1} = \frac{St - 0,10}{1,5 - d/b} \approx \frac{St - 0,10}{1,5 - 1,25} \quad \Rightarrow \quad St = 0,105 .$$

Extrapolation for Strouhal numbers as function of d/b are not allowed.

example-end

EXAMPLE 30-B- Criteria for vortex shedding: critical wind velocity - **test2**

Given: Find the critical wind velocity for bending vibration mode *i* (and the critical wind velocity for ovalling vibration mode *i* of cylindrical shells) for $St = 0,18$ [-]. Assume a natural frequency of the considered flexural mode i (of the ovalling mode i of the shell) equal to 1,5 Hz. The reference width of the cross-section (the outer shell diameter) is $0,6$ m.

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A131:N131-A164:N164*].

Solution: The critical wind velocity for bending vibration mode i is defined as the wind velocity at which the frequency of vortex shedding equals the natural frequency (mode *i*) of the structure or the structural:

$$v_{crit,i} = \frac{b \cdot n_{i,y}}{St} = \frac{0,6 \cdot 1,5}{0,18} = 5,00 \text{ m/s} .$$

The critical wind velocity for ovalling vibration mode *i* of cylindrical shells is defined as the wind velocity at which two times of the frequency of vortex shedding equals a natural frequency of the ovalling mode *i* of the cylindrical shell:

$$v_{crit,\,i} = \frac{b \cdot n_{i,\,0}}{2 \cdot St} = \frac{0,6 \cdot 1,5}{2 \cdot 0,18} = 2,50\ m/s$$

where b is the outer shell diameter and $n_{i,\,0}$ is the natural frequency of the ovalling mode *i* of the shell.

example-end

EXAMPLE 30-C- Basic parameters for vortex shedding: Scruton number - **test3**

Given: Find the Scruton number and the Reynolds number at the critical wind velocity $v_{crit,\,i}$ for a structural element with an equivalent mass per unit length (mode *i*) equal to $m_{i,\,e} = 3000\ kg/m$ and a structural damping $\delta_s = 5\%$ (expressed by the logarithmic decrement). Assume a reference width of the cross-section at which resonant vortex shedding occurs equal to $b = 0,6\ m$.

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A169:N169-A209:N209*].

Solution: With an air density under vortex shedding conditions equal to $\rho = 1,226\ kg/m^3$, we have:

$$Sc = \frac{2 \cdot \delta_s \cdot m_{i,\,e}}{\rho \cdot b^2} = \frac{2 \cdot 0,05 \cdot 3000}{1,226 \cdot (0,6)^2} = 679,72\ [-].$$

Critical velocity (see Sec. E.1.3.1), (see previous example) $v_{crit,\,i} = 5,00\ m/s$:

$$Re(v_{crit,\,i}) = \frac{b \cdot v_{crit,\,i}}{\nu} = \frac{0,6 \cdot 5,00}{15 \times 10^{-6}} = 0,2 \times 10^{6} = 200000\ [-]$$

with $\nu = 15 \times 10^{-6}\ m^2/s$ (kinematic viscosity of the air).

example-end

EXAMPLE 30-D- Vortex shedding action: effect of vibrations - **test4**

Given: A structural element of vibrating mass per unit length $m(s) = 1500\ kg/m$ has a natural frequency (mode shape i) $n_{i,\,y} = 0,5\ Hz$. The mode shape of the structure (normalised to 1) at the point "s" with the maximum displacement is equal to $y_{F,\,max} = 50\ mm$. Find the inertia force $F_w(s)$ per unit length acting perpendicular to the wind direction at location "s" on the structural element.

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A169:N169-A209:N209*].

Solution: Mode shape of the structure normalised to 1 at the point "s" with the maximum displacement (say): $\Phi_{i,y}(s) = 1,00$.

From Eq. (E.6):

$$F_w(s) = m(s) \cdot (2\pi \cdot n_{i,y})^2 \cdot \Phi_{i,y}(s) \cdot y_{F,max} = 1500 \cdot (2\pi \cdot 0,5)^2 \cdot 1,00 \cdot 0,05 = 740\ \mathrm{N/m}.$$

example-end

EXAMPLE 30-E- Calculation of the cross wind amplitude: Approach 1 - **test5**

Given: Find the largest displacement $y_{F,max}$ of the cross wind amplitudes for: $St = 0,18\ [-]$, $Sc = 679\ [-]$. Assume an effective correlation length factor (given in E.1.5.2.4) equal to $K_w = 0,6\ [-]$ and a Reynolds number $Re(v_{crit,i}) = 7000000\ [-]$ for a circular cross-section with $b = 60\ \mathrm{cm}$. The mode shape factor (given in E.1.5.2.5) is $K = 0,13\ [-]$.

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A235:N235-A256:N256*].

Solution: From Table E.2 and Figure E.2, for circular cross-section and $Re = 7 \times 10^6\ [-]$, we have:

$$\frac{0,3-0,2}{\log(10\times10^6)-\log(5\times10^6)} = \frac{c_{lat,0}-0,2}{\log(Re)-\log(5\times10^6)} \quad \Rightarrow \quad c_{lat,0} = 0,249\ [-].$$

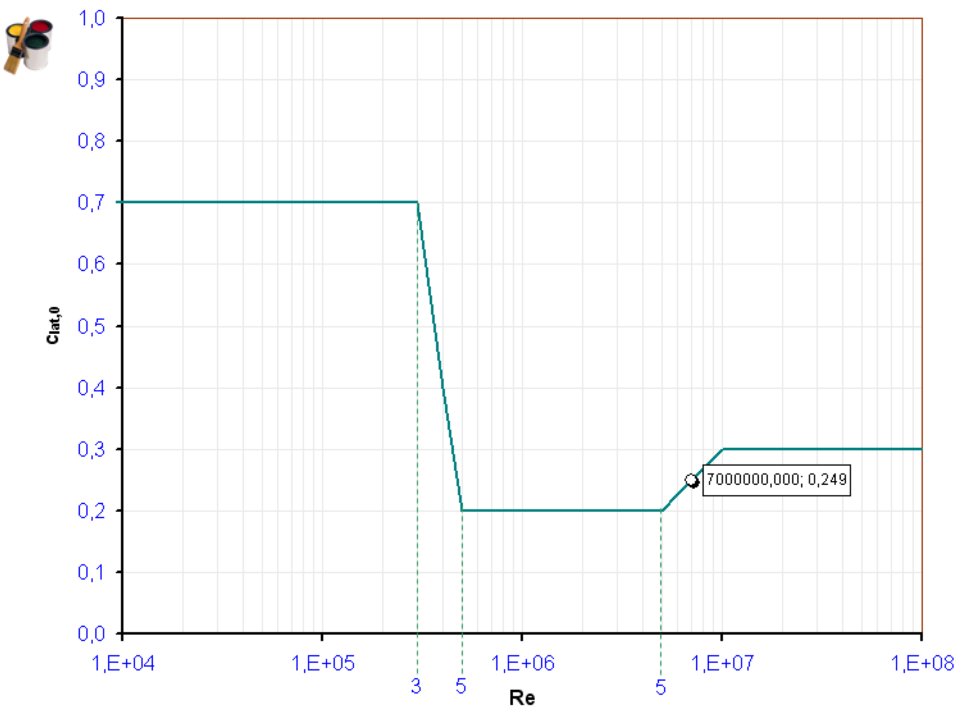


Figure 30.5 From Figure E.2 - Basic value of the lateral force coefficient $c_{lat,0}$ versus Reynolds number $Re(v_{crit,i})$ for circular cylinders, see E.1.3.4.

Assuming (say) $v_{crit,i}/v_{m,Lj} = 1,0$, from Table E.3 with $0,83 \le v_{crit,i}/v_{m,Lj} < 1,25$, we find:

$$c_{lat} = \left[3 - 2,4 \cdot \frac{v_{crit,i}}{v_{m,Lj}}\right] \cdot c_{lat,0} = [3 - 2,4 \cdot 1,0] \cdot 0,249 = 0,149\ [-].$$

From Expression (E.7), using the given numerical data, we get:

$$\frac{y_{F,max}}{b} = \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_w \cdot c_{lat} = \frac{1}{(0,18)^2} \cdot \frac{1}{679} \cdot 0,13 \cdot 0,6 \cdot 0,149 = 0,000528.$$

Therefore: $y_{F,max} = b \cdot 0,000528 = 0,60 \cdot 0,00051 = 0,00032\ m \approx 0,3\ mm$.

example-end

EXAMPLE 30-F- Calculation of the cross wind amplitude: correlation length - **test6**

Given: Find the effective correlation length $L_{j=1}$ for a vibration amplitude ($j = 1$) equal to $y_F(s_1) = 5\ cm$. Assume a width of the structure (length of the surface perpendicular to the wind direction) equal to $b = 1,20\ m$ (and then equal to $b = 0,40\ m$)

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A404:N404-A459:N459*].

Solution: We have: $y_F(s_1)/b = 0,05/1,2 = 0,0417$. From Table E.4, for $y_F(s_1)/b \le 1,0$, we get:

$L_1/b = 6,0$.

For $y_F(s_1)/b = 0,05/0,4 = 0,125$, from Table E.4 for $0,1 \le y_F(s_1)/b \le 0,6$, we get:

$L_1/b = 4,8 + 12 \cdot y_F(s_1)/b = 4,8 + 12 \cdot 0,125 = 6,30 \quad \rightarrow \quad L_1 = b \cdot 6,30 = 2,52\ m$.

example-end

EXAMPLE 30-G- Calculation of the cross wind amplitude: correlation length factor - **test7**

Given: Find the correlation length factors with $n = 1$ and $m = 1$ for the three simple structures in Table E.5. Assume $L_j/b = 6,00\ [-]$ for $b = 1,20\ m$ and $L_j/b = 6,30\ [-]$ for $b = 0,40\ m$ (see previous example). Length of the structure between two nodes (for cantilevered structures it is equal to the height of the structure): $l = 20\ m$. Assume $b = 0,4\ m$ for the vertical cantilever beam and $b = 1,20\ m$ for the horizontal beams.

[*Reference sheet:* Annex E_(a)]-[*Cell-Range: A465:N465-A510:N510*].

Solution: **Case a)** Cantilever, with $\lambda = l/b = 20/0,4 = 50,0\ [-]$ and $L_j/b = 6,30$:

$$K_w = \frac{3L_j/b}{\lambda} \cdot \left[1 - \frac{L_j/b}{\lambda} + \frac{1}{3} \cdot \left(\frac{L_j/b}{\lambda}\right)^2\right] = \frac{3 \cdot 6,30}{50} \cdot \left[1 - \frac{6,30}{50} + \frac{1}{3} \cdot \left(\frac{6,30}{50}\right)^2\right] = 0,332.$$

Case b) Simply supported beam spanning $l = 20\ m$, with

$\lambda = l/b = 20/1,20 = 16,67$ [-] and $L_j/b = 6,00$:

$$K_w = \cos\left[\frac{\pi}{2}\cdot\left(1-\frac{L_j/b}{\lambda}\right)\right] = \cos\left[\frac{\pi}{2}\cdot\left(1-\frac{6,00}{16,67}\right)\right] = 0,536 < 0,6\,.$$

Case c) Horizontal beam held rigidly at each end spanning $l = 20$ m, with

$\lambda = l/b = 20/1,20 = 16,67$ [-] and $L_j/b = 6,00$:

$$K_w = \frac{L_j/b}{\lambda} + \frac{1}{\pi}\cdot\sin\left[\pi\cdot\left(1-\frac{L_j/b}{\lambda}\right)\right] = \frac{6,00}{16,67} + \frac{1}{\pi}\cdot\sin\left[\pi\cdot\left(1-\frac{6,00}{16,67}\right)\right] = 0,648 > 0,6\,.$$

Actual value to be used in calculations (see Expression (E.8)): $K_w = 0,6$ [-].

example-end

30.5 References [Section 30]

EN 1991-1-4:2005/A1:2010. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, April 2010.

EN 1991-1-4:2005. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2005 (DAV).

Manual for the design of building structures to Eurocode 1 and Basis of Structural Design April 2010. © 2010 The Institution of Structural Engineers.

Eurocode Load Combinations for Steel Structures. The British Constructional Steelwork Association Limited. BCSA Publication No. 53/10. December 2010.

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Section 31

EN 1991-1-4 Annex E [from Sec. E.1.5.2.6 to Sec. E.4.3]

31.1 Calculation of the cross wind amplitude: number of load cycles

The number of load cycles N caused by vortex excited oscillation is given by expression:

$$N = 2T \cdot n_y \cdot \varepsilon_0 \cdot \left(\frac{v_{crit}}{v_0}\right)^2 \cdot \exp\left[-\left(\frac{v_{crit}}{v_0}\right)^2\right] \qquad \textbf{(Eq. 31-1)}$$

where:

- n_y is the natural frequency of cross-wind mode $[Hz]$. Approximations for n_y are given in Annex F
- v_{crit} is the critical wind velocity $[m/s]$ given in E.1.3.1
- v_0 is the 20% of the characteristic mean wind velocity as specified in Sec. 4.3.1(1)[(1)]
- T is the life time in seconds, which is equal to $3,2 \times 10^7$ multiplied by the expected lifetime in years
- ε_0 is the bandwidth factor describing the band of the wind velocities with vortex-induced vibrations.[(2)]

31.2 Vortex resonance of vertical cylinders in a row or grouped arrangement

For circular cylinders in a row or grouped arrangement with or without coupling (see Figure E.4) vortex excited vibrations may occur. The maximum deflections of oscillation can be estimated by Expression (E.7) and the calculation procedure

(1) v_0 is $\sqrt{2}$ times the modal value of the Weibull probability distribution assumed for the wind velocity $[m/s]$.

(2) The bandwidth factor ε_0 is in the range $0,1 \div 0,3$. It may be taken as $\varepsilon_0 = 0,3$.

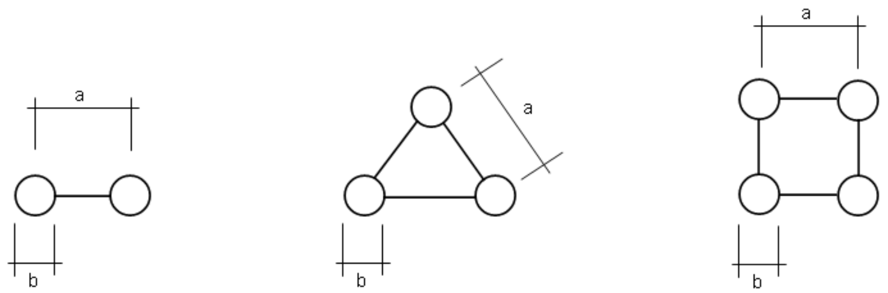


Figure 31.1 From Figure E.4 - In-line and grouped arrangements of cylinders.

given in E.1.5.2 with the modifications given by the following expressions. For in-line, free standing circular cylinders without coupling:

$$c_{lat} = 1,5 \cdot c_{lat(single)} \text{ for } 1 \le \frac{a}{b} \le 10$$

$$c_{lat} = c_{lat(single)} \text{ for } \frac{a}{b} \ge 15$$

linear interpolation for $10 < \frac{a}{b} \le 15$ **(Eq. 31-2)**

$$St = 0,1 + 0,085 \cdot \log\left(\frac{a}{b}\right) \text{ for } 1 \le \frac{a}{b} \le 9$$

$$St = 0,18 \text{ for } \frac{a}{b} > 9.$$

where:

$c_{lat(single)} = c_{lat}$ as given in Table E.3. For coupled cylinders:

$$c_{lat} = K_{iv} \cdot c_{lat(single)} \text{ for } 1,0 \le \frac{a}{b} \le 3,0 \qquad \textbf{(Eq. 31-3)}$$

where:

- K_{iv} is the interference factor for vortex shedding (Table E.8)
- St is the Strouhal number (given in Table E.8)
- Sc is the Scruton number (given in Table E.8).

For coupled cylinders with $a/b > 3,0$ specialist advice is recommended.

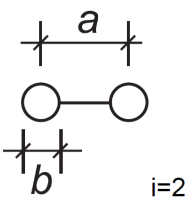
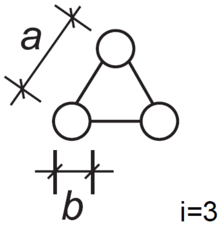
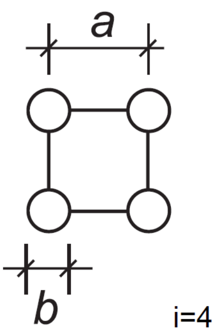
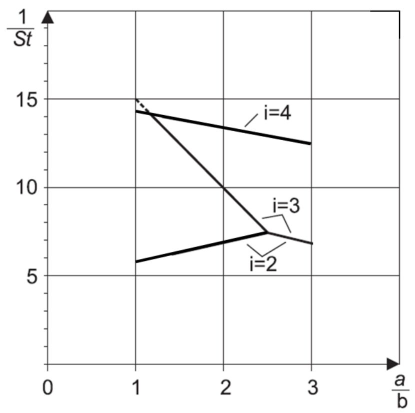
Coupled cylinders	Scruton number Sc (see Eq. E.4)			
	a/b = 1	a/b ≥ 2	a/b ≤ 1,5	a/b ≥ 2,5
	K_{iv} = 1,5	K_{iv} = 1,5	a_G = 1,5	a_G = 3,0
	K_{iv} = 4,8	K_{iv} = 3,0	a_G = 6,0	a_G = 3,0
	K_{iv} = 4,8	K_{iv} = 3,0	a_G = 1,0	a_G = 2,0
	Linear interpolation			
	 Reciprocal Strouhal numbers of coupled cylinders with in-line and grouped arrangements.			
NOTE: extrapolation for the factor a_G as function of d/b are not allowed.				

Table 31.1 From Table E.8 - Data for estimation of cross-wind response of coupled cylinders at in-line and grouped arrangements.

31.3 Approach 2, for the calculation of the cross wind amplitudes

The characteristic maximum displacement at the point with the largest movement is given in expression:

$$y_{max} = \sigma_y \cdot k_p \qquad \textbf{(Eq. 31-4)}$$

where:

- σ_y is the standard deviation of the displacement
- k_p is the peak factor (see Eq. 31-8 below).

The solution of the Eq. 31-4 is given by the following expression:

$$\left(\frac{\sigma_y}{b}\right)^2 = c_1 + \sqrt{c_1^2 + c_2} \qquad \textbf{(Eq. 31-5)}$$

where the constants c_1 and c_2 are given by:

$$c_1 = \frac{a_L^2}{2} \cdot \left(1 - \frac{Sc}{4\pi \cdot K_a}\right) \qquad \textbf{(Eq. 31-6)}$$

$$c_2 = \frac{\rho \cdot b^2}{m_e} \cdot \frac{a_L^2}{K_a} \cdot \frac{C_c^2}{St^4} \cdot \frac{b}{h} \qquad \textbf{(Eq. 31-7)}$$

where:

- C_c is the aerodynamic constant dependent on the cross-sectional shape, and for a circular cylinder also dependent on the Reynolds number Re as defined in E.1.3.4(1), given in Table E.6.
- K_a is the aerodynamic damping parameter as given in E.1.5.3(4)
- a_L is the normalised limiting amplitude giving the deflection of structures with very low damping, given in Table E.6
- St is the Strouhal number given in Table E.1
- ρ is the air density under vortex shedding conditions
- m_e is the effective mass per unit length, given in F.4(1)
- h, b are the height and width of structure respectively. For structures with varying width, the width at the point with largest displacements is used.

The aerodynamic damping constant K_a decreases with increasing turbulence intensity. For a turbulence intensity of 0%, the aerodynamic damping constant may be taken as $K_a = K_{a,max}$, which is given in Table E.6. For a circular cylinder and square cross-section the constants C_c, $K_{a,max}$ and a_L are given in Table E.6.

The peak factor k_p should be determined by the following expression:

$$k_p = \sqrt{2} \cdot \left\{1 + 1,2 \cdot \arctan\left[0,75 \cdot \left(\frac{Sc}{4\pi \cdot K_a}\right)^4\right]\right\} \qquad \textbf{(Eq. 31-8)}$$

Constants[(a)]	Circular cylinder $Re \leq 10^5$	Circular cylinder $Re = 5 \times 10^5$	Circular cylinder $Re \geq 10^6$	Square cross-section
C_c	0,02	0,005	0,01	0,04
$K_{a,\max}$	2	0,5	1	6
a_L	0,4	0,4	0,4	0,4

Table 31.2 From Table E.6 - Constants for determination of the effect of vortex shedding.

(a). For circular cylinders the constants C_s and $K_{a,max}$ are assumed to vary linearly with the logarithm of the Reynolds number for $10^5 < Re < 5 \times 10^5$ and for $5 \times 10^5 < Re < 10^6$, respectively.

The number of load cycles may be obtained from E.1.5.2.6 using a bandwidth factor of $\varepsilon_0 = 0,15$.

31.4 Galloping

31.4.1 Onset wind velocity

Galloping is a self-induced vibration of a flexible structure in cross wind bending mode. Non circular cross sections including L-, I-, U- and T-sections are prone to galloping. Ice may cause a stable cross section to become unstable. Galloping oscillation starts at a special onset wind velocity v_{CG} and normally the amplitudes increase rapidly with increasing wind velocity. The onset wind velocity of galloping, v_{CG}, is given in expression:

$$v_{CG} = \frac{2Sc}{a_G} \cdot n_{1,y} \cdot b \qquad \textbf{(Eq. 31-9)}$$

where:

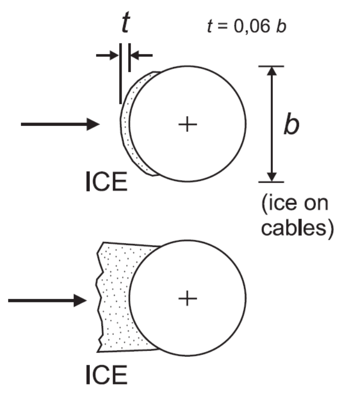
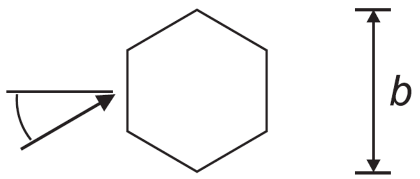
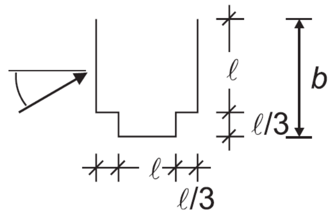
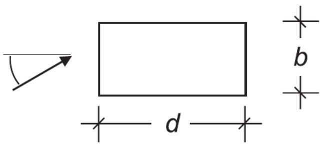
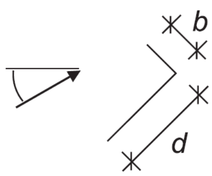
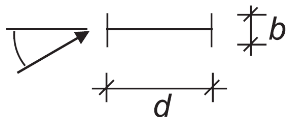
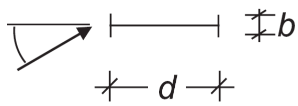
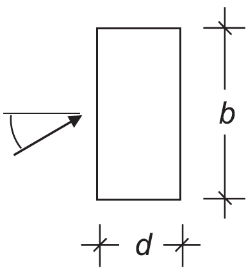
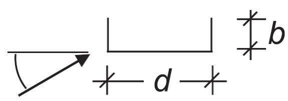
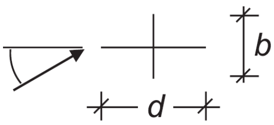
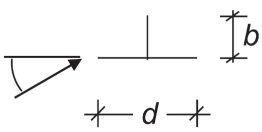
- Sc is the Scruton number as defined in E.1.3.3(1)
- $n_{1,y}$ is the cross-wind fundamental frequency of the structure[(3)]
- b is the width of the structural element/structure as defined in Table E.7 below
- a_G is the factor of galloping instability (see Table E.7); if no factor of galloping instability is known, $a_G = 10$ may be used.

It should be ensured that:

$$v_{CG} > 1,25 \cdot v_m(z) \qquad \textbf{(Eq. 31-10)}$$

where $v_m(z)$ is the mean wind velocity as defined in Expression (4.3)[(4)] and calculated at the height z, where galloping process is expected, likely to be the

(3) Approximations of $n_{1,y}$ are given in Annex F, Sec. F.2.

Cross-section		Factor of galloping instability a_G	Cross-section		Factor of galloping instability a_G
		1,0			1,0
					4
 linear interpolation	d/b = 2	2		d/b = 2	0,7
	d/b = 1,5	1,7		d/b = 2,7	5
	d/b = 1	1,2		d/b = 5	7
 linear interpolation	d/b = 2/3	1		d/b = 3	7,5
	d/b = 1/2	0,7		d/b = 3/4	3,2
	d/b = 1/3	0,4		d/b = 2	1

NOTE: extrapolation for the factor a_G as function of d/b are not allowed.

Table 31.3 From Table E.7 - Factor of galloping instability a_G.

(4) See Section 4.3.1.

point of maximum amplitude of oscillation. If the critical vortex shedding velocity v_{crit} is close to the onset wind velocity of galloping v_{CG}:

$$0,7 < \frac{v_{CG}}{v_{crit}} < 1,5 \qquad \textbf{(Eq. 31-11)}$$

interaction effects between vortex shedding and galloping are likely to occur. In this case specialist advice is recommended.

31.4.2 Classical galloping of coupled cylinders

For coupled cylinders (see Figure 31.1) classical galloping may occur. The onset velocity for classical galloping of coupled cylinders, v_{CG}, may be estimated by expression:

$$v_{CG} = \frac{2Sc}{a_G} \cdot n_{1,y} \cdot b \qquad \textbf{(Eq. 31-12)}$$

where Sc, a_G and b are given in Table E.8 and $n_{1,y}$ is the natural frequency of the bending mode (see Annex F, Sec. F.2). It should be ensured that:

$$v_{CG} > 1,25 \cdot v_m(z) \qquad \textbf{(Eq. 31-13)}$$

where $v_m(z)$ is the mean wind velocity as defined in Expression (4.3), calculated at the height z, where the galloping excitation is expected, that is likely to be the point of maximum amplitude of oscillation.

31.4.3 Interference galloping of two or more free standing cylinders

Interference galloping is a self-excited oscillation which may occur if two or more cylinders are arranged close together without being connected with each other. If the angle of wind attack is in the range of the critical wind direction β_k and if $a/b < 3$ (see Figure E.5), the critical wind velocity, v_{CIG}, may be estimated by:

$$v_{CIG} = 3,5 \cdot n_{1,y} \cdot b \cdot \sqrt{\frac{\frac{a}{b} \cdot Sc}{a_{IG}}} \qquad \textbf{(Eq. 31-14)}$$

where:

- Sc is the Scruton number as defined in Sec. E.1.3.3(1)
- $a_{IG} = 3,0$ is the combine stability parameter
- $n_{1,y}$ is the fundamental frequency of cross-wind mode.(5)
- a is the spacing
- b is the diameter.

(5) Approximations are given in Annex F, Sec. F.2.

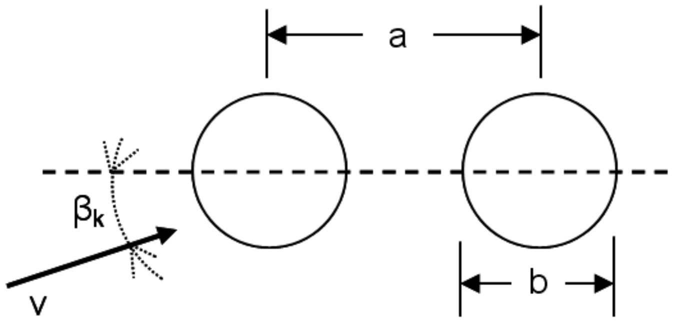


Figure 31.2 From Figure E.5 - Geometric parameters for interference galloping.

Interference galloping can be avoided by coupling the free-standing cylinders. In that case classical galloping may occur (see E.2.3).

31.5 Divergence and Flutter

31.5.1 Criteria for plate-like structures

Divergence and flutter are instabilities that occur for flexible plate-like structures, such as signboards or suspension-bridge decks, above a certain threshold or critical wind velocity. The instability is caused by the deflection of the structure modifying the aerodynamics to alter the loading. Divergence and flutter should be avoided.

To be prone to either divergence or flutter, the structure satisfies all of the three criteria given below:

1. the structure, or a substantial part of it, has an elongated cross-section (like a flat plate) with b/d less than 0,25 (see Figure E.6)
2. the torsional axis is parallel to the plane of the plate and normal to the wind direction, and the centre of torsion is at least $d/4$ downwind of the windward edge of the plate, where d is the inwind depth of the plate measured normal to the torsional axis. This includes the common cases of torsional centre at geometrical centre, i.e. centrally supported signboard or canopy, and torsional centre at downwind edge, i.e. cantilevered canopy
3. the lowest natural frequency corresponds to a torsional mode, or else the lowest torsional natural frequency is less than 2 times the lowest translational natural frequency.

The criteria should be checked in the order given (easiest first) and if any one of the criteria is not met, the structure will not be prone to either divergence or flutter.

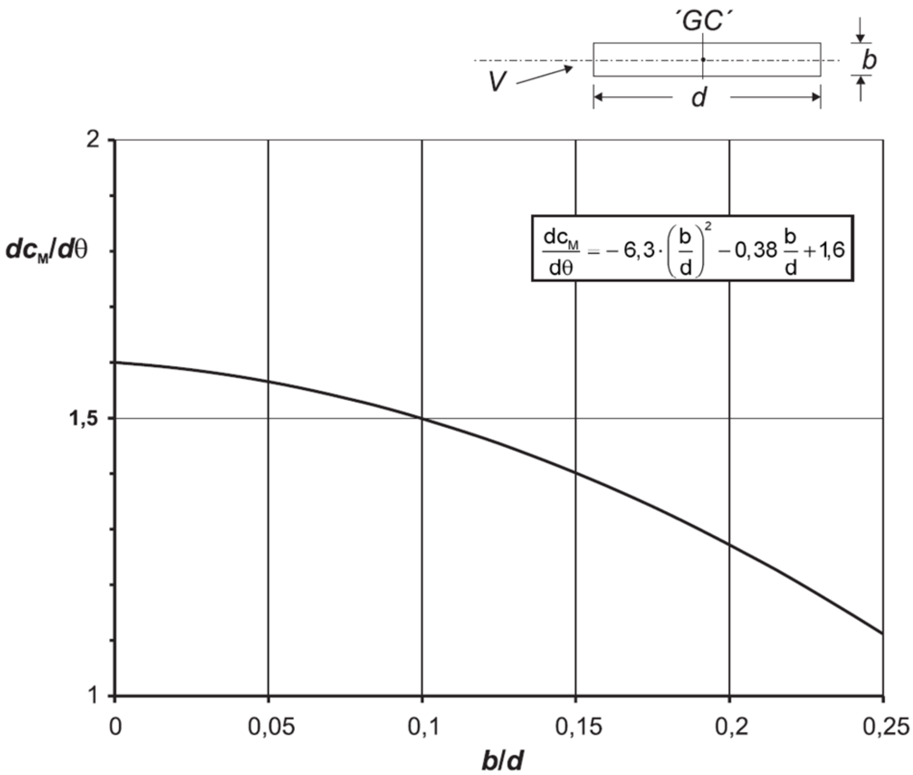


Figure 31.3 From Figure E.6 - Rate of change of aerodynamic moment coefficient, $dc_M/d\theta$, with respect to geometric centre "GC" for rectangular section".

31.5.2 Divergency velocity

The critical wind velocity for divergence is given in expression:

$$v_{div} = \left(\frac{2k_\theta}{\rho \cdot d^2 \cdot \frac{dc_M}{d\theta}}\right)^{\frac{1}{2}} \qquad \textbf{(Eq. 31-15)}$$

where:

- k_θ is the torsional stiffness $[Nm/rad]$
- c_M is the aerodynamic moment coefficient (see Eq. E.25)
- $dc_M/d\theta$ is the rate of change of aerodynamic moment coefficient with respect to rotation about the torsional centre, θ is expressed in radians
- ρ is the density of the air (see Sec. 4.5)
- d is the in wind depth (chord) of the structure (see Figure E.6)
- b is the width as defined in Figure E.6.

Values of $dc_M/d\theta$ measured about the geometric centre of various rectangular sections are given in Figure E.6.

It should be ensured that:

$$v_{div} > 2 \cdot v_m(z_s) \qquad \textbf{(Eq. 31-16)}$$

where $v_m(z_s)$ is the mean wind velocity as defined in Eq. (4.3) at height z_s (defined in Figure 6.1).

31.6 Verification tests

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— Splash

— Annex E_(b).

EXAMPLE 31-A- Calculation of the cross wind amplitude: number of load cycles - **test1**

Given: Find the number of load cycles N caused by vortex excited oscillation for:

– a natural frequency of cross-wind mode $n_y = 4,50\ \mathrm{Hz}$

– a critical wind velocity $v_{crit} = 5,5\ \mathrm{m/s}$

– a life time of the structure equal to $t = 50$ years and

– a bandwidth factor $\varepsilon_0 = 0,3$.

Assume $v_0 = 0,20 \cdot v_m = 5\ \mathrm{m/s}$ where $v_m = 25\ \mathrm{m/s}$ is the characteristic mean wind velocity as specified in 4.3.1(1).

[*Reference sheet:* Annex E_(b)]-[*Cell-Range: A1:N1-A25:N25*].

Solution: From Eq. (E.10), substituting the given numerical data, we have:

$N = 2T \cdot n_y \cdot \varepsilon_0 \cdot \left(\frac{v_{crit}}{v_0}\right)^2 \cdot \exp\left[-\left(\frac{v_{crit}}{v_0}\right)^2\right]$. With $T = (3,2\times10^7) \cdot 50 = 1,6\times10^9\ \mathrm{s}$, we get:

$$N = 2 \cdot (1,6\times10^9) \cdot 4,50 \cdot 0,3 \cdot \left(\frac{5,5}{5,0}\right)^2 \cdot \exp\left[-\left(\frac{5,5}{5,0}\right)^2\right] = 1,5587\times10^9 \approx 1,6\ \text{billion}.$$

It means a load cycles per second equal to:

$$\frac{N}{T} = \frac{1,5587\times10^9}{1,6\times10^9} \approx 1.$$

NOTE The National Annex may specify the minimum value of N. The recommended value is $N \geq 10^4$.

example-end

EXAMPLE 31-B- Vortex resonance of vertical cylinders in a row or grouped arrangement - **test2**

Given: Estimate the maximum deflection of oscillation of a (**Case a**) free standing and (**Case b**) in-line/grouped arrangements of cylinders (see Figure E.4) with $a/b = 1,70$ and $b = 0,50\ m$. For $c_{lat(single)} = 0,20\ [-]$, assume $Sc = 120\ [-]$ for in-line, free standing circular cylinders without coupling and $Sc = 400\ [-]$ for "coupled". The effective correlation length factor K_w (given in E.1.5.2.4) is equal to 0,60 and 0,80 respectively for "in-line" and "coupled". Similarly, the mode shape factor K is equal (say) to 0,13 and 0,15 respectively.

[*Reference sheet:* Annex E_(b)]-[*Cell-Range: A29:N29-A169:N169*].

Solution: **Case a)** In-line, free standing circular cylinders without coupling:

for $1 \le a/b \le 10$: $c_{lat} = 1,5 \cdot c_{lat(single)} = 1,5 \cdot 0,20 = 0,30\ [-]$.

for $1 \le a/b \le 9$: $St = 0,1 + 0,085 \cdot \log(a/b) = 0,1 + 0,085 \cdot \log(1,70) = 0,120$.

Case b) For coupled cylinders (with i = 2-3-4):

for $1 \le a/b \le 3$: $c_{lat} = K_{iv} \cdot c_{lat(single)} = 3,54 \cdot 0,20 = 0,71\ [-]$, having considered for K_{iv} the linear interpolation between (say) the point A(1; 4,8) and B(2; 3,0) in Table E.8:

$$\frac{4,8 - 3,0}{2 - 1} = \frac{K_{iv} - 3,0}{2 - a/b} \quad \Rightarrow \quad \frac{4,8 - 3,0}{2 - 1} = \frac{K_{iv} - 3,0}{1 - 1,70} \quad \rightarrow \quad K_{iv} = 3,54\ [-].$$

Entering Table E.8 for $a/b = 1,70$ we obtain $1/St = 6 \Rightarrow St \approx 0,170\ [-]$.

Case a): From Eq. (E.7):

$$\frac{y_{F,max}}{b} = \frac{1}{St^2 \cdot Sc} \cdot K \cdot K_w \cdot c_{lat} = \frac{1}{(0,120)^2 \cdot (120)} \cdot (0,13) \cdot (0,60) \cdot (0,30) \approx 0,0135$$

Therefore: $y_{F,max} = 0,0135 \cdot b = 0,0135 \cdot 0,50 = 0,00677 = 6 \div 7\ mm$.

Case b):

$$\frac{y_{F,max}}{b} = \frac{1}{St^2 \cdot Sc} \cdot K \cdot K_w \cdot c_{lat} = \frac{1}{(0,170)^2 \cdot (400)} \cdot (0,15) \cdot (0,80) \cdot (0,71) \approx 0,0073$$

Therefore: $y_{F,max} = 0,0135 \cdot b = 0,0073 \cdot 0,50 = 0,0037 = 3 \div 4\ mm$.

example-end

EXAMPLE 31-C- Approach 2, for the calculation of the cross wind amplitudes- **test3**

Given: Estimate the characteristic maximum displacement at the point with the largest movement of a structure with a circular cylinder shape. Assume:

– height of the structure: $h = 6,0\ m$

– width of the structure (at the point with largest displacements): $b = 0,8\ m$

– air density under vortex shedding conditions: $\rho = 1,25\ kg/m^3$

– effective mass per unit length (given in F.4 (1)): $m_e = 1000\ kg/m$

– Strouhal number (given in Table E.1): $St = 0,180$ [-]

– Scruton number (given in E.1.3.3): $Sc = 125$ [-]

– Reynolds number (at the point with largest displacements): $Re = 2,5\times10^5$ [-].

[*Reference sheet:* Annex E_(b)]-[*Cell-Range: A172:N172-A255:N255*].

Solution: Entering Table E.6 with $Re = 2,5\times10^5$ [-], for circular cylinders assuming C_c and $K_{a,\max}$ to vary linearly with the logarithm of the Reynolds number for $10^5 \le Re \le 5\times10^5$, we get:

$$C_c = 0,005 + \frac{[\log(5\times10^5) - \log(Re)]}{\log(5\times10^5) - \log(10^5)} \cdot (0,02 - 0,005) = 0,0115$$

$$K_a = K_{a,\max} = 0,5 + \frac{[\log(5\times10^5) - \log(Re)]}{\log(5\times10^5) - \log(10^5)} \cdot (2 - 0,5) = 1,1460 .$$

Therefore, with $a_L = 0,4$, we have:

$$c_1 = \frac{a_L^2}{2} \cdot \left(1 - \frac{Sc}{4\pi \cdot K_a}\right) = \frac{(0,4)^2}{2} \cdot \left(1 - \frac{125}{4\pi \cdot 1,1460}\right) = -0,614$$

$$c_2 = \frac{\rho \cdot b^2}{m_e} \cdot \frac{a_L^2}{K_a} \cdot \frac{C_c^2}{St^4} \cdot \frac{b}{h} = \frac{1,25 \cdot (0,8)^2}{1000} \cdot \frac{(0,4)^2}{(1,1460)} \cdot \frac{(0,0115)^2}{(0,180)^2} \cdot \frac{(0,8)}{(6,0)} \approx 1,9\times10^{-6} .$$

From the expression:

$$\left(\frac{\sigma_y}{b}\right)^2 = c_1 + \sqrt{c_1^2 + c_2} = -0,614 + \sqrt{(-0,614)^2 + 1,9\times10^{-6}} \approx 1,55\times10^{-6}$$

we find the standard deviation of the displacement:

$$\sigma_y/b = \sqrt{1,55\times10^{-6}} \approx 0,00124 \quad \Rightarrow \quad \sigma_y = 0,00124 \cdot 0,8 = 0,0099 .$$

The peak factor is given by the expression:

$$k_p = \sqrt{2} \cdot \left\{1 + 1,2 \cdot \arctan\left[0,75 \cdot \left(\frac{Sc}{4\pi \cdot K_a}\right)^4\right]\right\} = \sqrt{2} \cdot \left\{1 + 1,2 \cdot \arctan\left[0,75 \cdot \left(\frac{125}{4\pi \cdot 1,1460}\right)^4\right]\right\}$$

$$k_p = 4,08 .$$

The characteristic maximum displacement at the point with the largest movement is given in expression: $y_{max} = \sigma_y \cdot k_p = 0,00099 \cdot 4,08 = 0,0040\ m = 4\ mm$.

example-end

EXAMPLE 31-D- Galloping: Onset wind velocity - **test4**

Given: Find the onset wind velocity of galloping for a rectangular cross-section with $b = 0,30\ m$, $d = 0,60\ m$ and for:

– a Scruton number as defined in E.1.3.3(1): $Sc = 125$ [-]

– a cross-wind fundamental frequency of the structure: $n_{1,y} = 0,5\ Hz$ (see Sec. F.2)

– a width of the structure (as defined in Table E.7): $b = 0,30\ m$

divergence for a density of air $\rho = 1,25\ \mathrm{kg/m^3}$ and for a mean wind velocity (as defined in Expression 4.3) at height z_s (defined in Figure 6.1) equal to $20,00\ \mathrm{m/s}$.

[*Reference sheet:* Annex E_(b)]-[*Cell-Range: A555:N555-A628:N628*].

Solution: From Figure E.6 with $b/d = 0,1875$:

$$\frac{dc_M}{d\theta} = -6,3\cdot\left(\frac{b}{d}\right)^2 - 0,38\cdot\frac{b}{d} + 1,6 = -6,3\cdot(0,1875)^2 - 0,38\cdot(0,1875) + 1,6 = 1,307.$$

From Expression (E.24):

$$v_{div} = \left(\frac{2k_\theta}{\rho\cdot d^2\cdot\frac{dc_M}{d\theta}}\right)^{\frac{1}{2}} = \left(\frac{2\cdot 1000}{1,25\cdot(0,80)^2\cdot 1,307}\right)^{1/2} = 43,73\ \mathrm{m/s}.$$

It should be ensured that: $v_{div} > 2\cdot v_m(z_s)$.

Substituting the given numerical data into expression above we find:

$v_{div} = 43,73\ \mathrm{m/s} > 2\cdot 20,00\ \mathrm{m/s}$ **[Satisfactory].**

example-end

31.7 References [Section 31]

EN 1991-1-4:2005/A1:2010. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, April 2010.

EN 1991-1-4:2005. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2005 (DAV).

Manual for the design of building structures to Eurocode 1 and Basis of Structural Design April 2010. © 2010 The Institution of Structural Engineers.

Eurocode Load Combinations for Steel Structures. The British Constructional Steelwork Association Limited. BCSA Publication No. 53/10. December 2010.

Section 32 EN 1991-1-4 Annex F

32.1 Dynamic characteristics of structures

Calculation procedures recommended in this section assume that structures possess linear elastic behaviour and classical normal modes. Dynamic structural properties are therefore characterised by:

- — natural frequencies
- — modal shapes
- — equivalent masses
- — logarithmic decrements of damping.

Natural frequencies, modal shapes, equivalent masses and logarithmic decrements of damping should be evaluated, theoretically or experimentally, by applying the methods of structural dynamics.

32.2 Fundamental frequency

FLEXURAL FREQUENCY N_1. For cantilevers with one mass at the end a simplified expression to calculate the fundamental flexural frequency n_1 of structures is given by expression:

$$n_1 = \frac{1}{2\pi}\sqrt{\frac{g}{x_1}} \qquad \textbf{(Eq. 32-17)}$$

where:

- $g = 9,81\ m/s^2$ is the acceleration of gravity
- x_1 is the maximum displacement due to self weight applied in the vibration direction.

The fundamental flexural frequency n_1 of multi-storey buildings with a height larger than 50 m can be estimated using expression:

$$n_1\ [Hz] = \frac{46}{h} \qquad \textbf{(Eq. 32-18)}$$

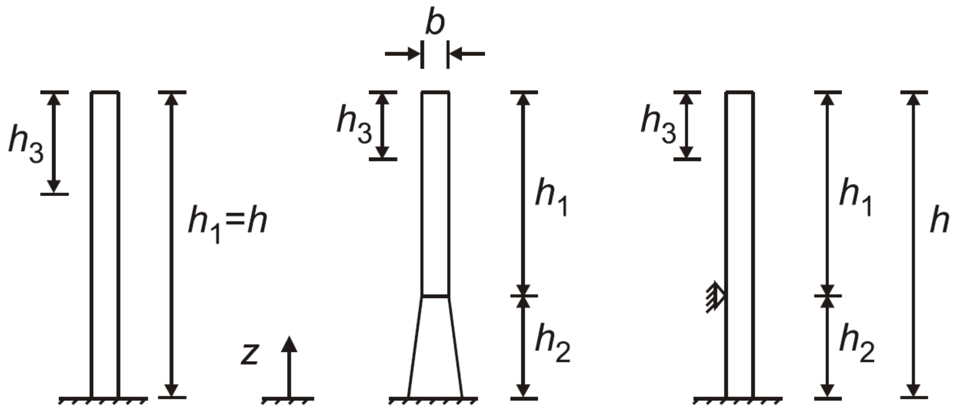


Figure 32.4 From Figure F.1 - Geometric parameters for chimneys.

where h is the height of the structure in meters.

Note The same expression may give some guidance for single-storey buildings and towers.

FLEXURAL FREQUENCY N_1. The fundamental flexural frequency n_1, of chimneys can be estimated by expression:

$$n_1\ [\mathrm{Hz}] = \frac{\varepsilon_1 \cdot b}{h_{eff}^2} \cdot \sqrt{\frac{W_s}{W_t}} \qquad \textbf{(Eq. 32-19)}$$

with $h_{eff} = h_1 + h_2/3$ and where:

- b is the top diameter of the chimney
- h_{eff} is the effective height of the chimney $[m]$, h_1 and h_2 are given in Figure F.1.
- W_s is the weight of structural parts contributing to the stiffness of the chimney
- W_t is the total weight of the chimney
- ε_1 is equal to 1000 for steel chimney, and 700 for concrete and masonry chimneys.

OVALLING FREQUENCY $N_{1,0}$. The fundamental (lowest) ovalling frequency $n_{1,0}$ of a long cylindrical shell without stiffening rings may be calculated using expression:

$$n_{1,0} = 0,492 \cdot \sqrt{\frac{t^3 \cdot E}{\mu_s \cdot (1 - \nu^2) \cdot b^4}} \qquad \textbf{(Eq. 32-20)}$$

where:

- $E\ [N/m^2]$ is Young's modulus (of the structural material)
- $t\ [m]$ is the shell tickness

- ν is Poisson ratio (of the structural material)
- μ_s is the mass of the shell per unit area $[kg/m^2]$
- $b\ [m]$ is the diameter of the shell.

VERTICAL BENDING FREQUENCY $n_{1,B}$. The frequency $n_{1,B}$ of a plate or box girder bridge may be approximately derived from expression:

$$n_{1,B} = \frac{K^2}{2\pi \cdot L^2} \cdot \sqrt{\frac{EI_b}{m}} \qquad \text{(Eq. 32-21)}$$

where:

- $L\ [m]$ is the length of the main span of the bridge
- $E\ [N/m^2]$ is Young's modulus (of the plate or girders bridge)
- $I_b\ [m^4]$ is the second moment of area of the full cross-section of the bridge for vertical bending at mid-span

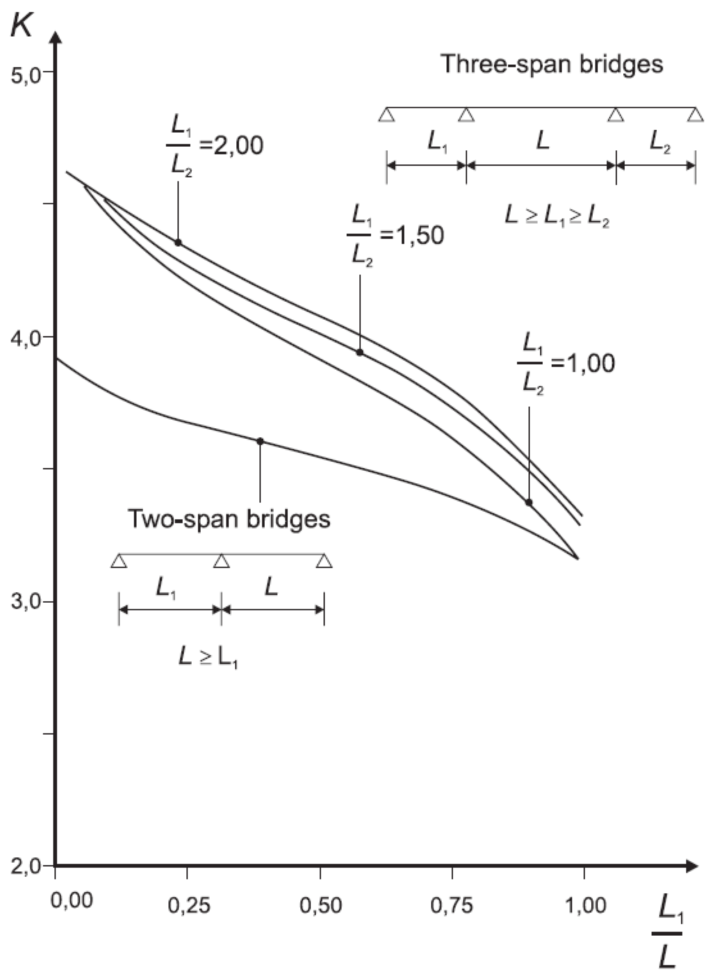


Figure 32.5 From Figure F.2 - Factor K used for the derivation of fundamental bending frequency.

- $m\ [kg/m]$ is the mass per unit length of the full cross-section at midspan (for dead and super-imposed dead loads)
- K is a dimensionless factor depending on span arrangement: for single span bridge:
  - $K = \pi$ if simply supported
  - $K = 3,9$ if propped cantilevered
  - $K = 4,7$ if fixed end supports

  in the case of more spans K is obtained from Figure F.2 above.

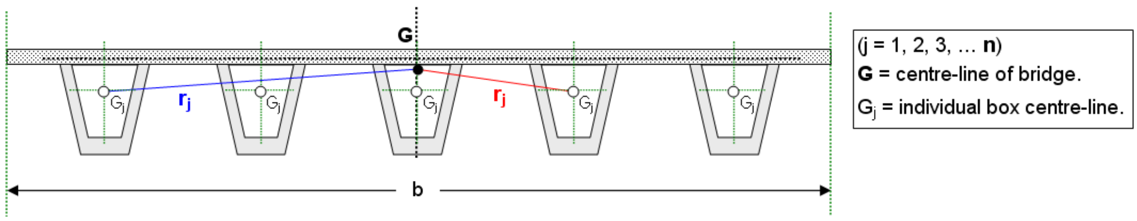
Note If the value of $\sqrt{EI_b/m}$ at the support exceeds twice the value at mid-span, or is less than 80% of the mid-span value, then the eq. 32-21 should not be used unless very approximate values are sufficient. The fundamental torsional frequency of plate girder bridges is equal to the fundamental bending frequency calculated from eq. 32-21, provided the average longitudinal bending inertia per unit width is not less than 100 times the average transverse bending inertia per unit length.

TORSIONAL FREQUENCY. The fundamental torsional frequency of a box girder bridge may be approximately derived from equation:

$$n_{1,T} = n_{1,B} \cdot \sqrt{P_1 \cdot (P_2 + P_3)} \qquad \textbf{(Eq. 32-22)}$$

with:

- $P_1 = \dfrac{mb^2}{I_p}$
- $P_2 = \dfrac{\sum r_j^2 \cdot I_j}{b^2 \cdot I_p}$
- $P_3 = \dfrac{L^2 \cdot \sum J_j}{2K^2 \cdot b^2 \cdot I_p \cdot (1+\nu)}$



LEGENDA

r_j is the distance of individual box centre-line from centre-line of bridge.

I_j is the second moment of mass per unit length of individual box for vertical bending at midspan, including an associated effective width of deck. $\sum$ represents summation over all the box girders in the cross-section.

m_d is the mass per unit length of the deck only, at mid-span.

I_{pj} is the mass moment of inertia of individual box at mid-span.

n is the number of the boxes.

m_j is the mass per unit length of individual box only, at mid-span, without associated portion of deck.

J_j is the torsion constant of individual box at mid-span (see Eq. F.12).

Figure 32.6 Cross-section of the bridge at mid-span.

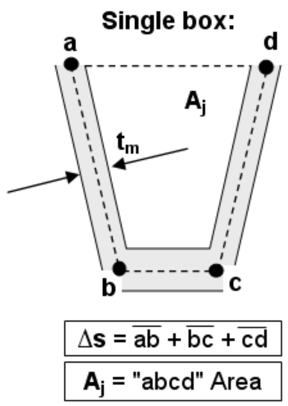


Figure 32.7 Cross-section single box at mid-span.

where:

- $n_{1,B}$ is the fundamental bending frequency in Hz
- b is the total width of the bridge
- m is the mass per unit length defined in F.2(5)
- ν is Poisson's ratio of girder material
- r_j is the distance of individual box centre-line from centre-line of bridge
- $I_j\ [kg \cdot m^2/m]$ is the second moment of mass per unit length of individual box for vertical bending at mid-span, including an associated effective width of deck
- $I_b\ [m^4]$ is the second moment of area of the full cross-section of the bridge for vertical bending at mid-span
- $I_p\ [kg \cdot m^2/m]$ is the second moment of mass per unit length of the full cross-section of the bridge at mid-span. It is described by equation:

$$I_p = \frac{m_d \cdot b^2}{12} + \sum(I_{pj} + m_j \cdot r_j^2) \qquad \textbf{(Eq. 32-23)}$$

where:

- m_d is the mass per unit length of the deck only, at mid-span
- I_{pj} is the mass moment of inertia (per unit length) of individual box at mid-span
- m_j is the mass per unit length of individual box only, at mid-span, without associated portion of deck.
- $J_j\ [m^4]$ is the torsion constant of individual box at mid-span. It is described by expression (F.12):

$$J_j = \frac{4A_j^2}{\oint \frac{ds}{t}} \qquad \textbf{(Eq. 32-24)}$$

where:

- A_j is the enclosed cell area at mid-span (see Figure 32.7 above)
- $\oint ds/t$ is the integral around box perimeter of the ratio length/thickness for each portion of box wall at mid-span.

Note Slight loss of accuracy may occur if the proposed Eq. 32-24 is applied to multibox bridges whose plan aspect ratio (= span/width) exceeds 6.

32.3 Fundamental mode shape

The fundamental flexural mode $\Phi_1(z)$ of buildings, towers and chimneys cantilevered from the ground may be estimated using expression:

$$\Phi_1(z) = \left(\frac{z}{h}\right)^{\zeta} \qquad \textbf{(Eq. 32-25)}$$

where:

- $\zeta = 0,6$ for slender frame structures with non load-sharing walling or cladding
- $\zeta = 1,0$ for buildings with a central core plus peripheral columns or larger columns plus shear bracings

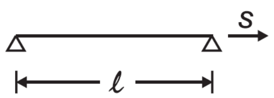
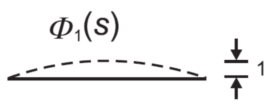
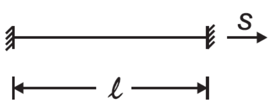
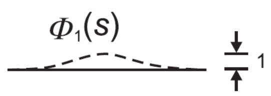
Scheme	Mode shape	$\Phi_1(s)$
		$\sin\left(\pi \cdot \frac{s}{l}\right)$
		$\frac{1}{2} \cdot \left[1 - \cos\left(2\pi \cdot \frac{s}{l}\right)\right]$

Table 32.4 From Table F.1 - Fundamental flexural vertical mode shape for simple supported and clamped structures and structural elements.

- $\zeta = 1,5$ for slender cantilever buildings and buildings supported by central reinforced concrete cores
- $\zeta = 2,0$ for towers and chimneys
- $\zeta = 2,5$ for lattice steel towers.

The fundamental flexural vertical mode $\Phi_1(s)$ of bridges may be estimated as shown in Table 32.4 above.

32.4 Equivalent mass

The equivalent mass per unit length m_e of the fundamental mode is given by:

$$m_e = \frac{\int_0^l m(s) \cdot \Phi_1^2(s)ds}{\int_0^l \Phi_1^2(s)ds} \approx \frac{\sum_{j=1}^{N} \Delta s_j \cdot \Phi_1^2(s_{j,m}) \cdot m(s_{j,m})}{\sum_{j=1}^{N} \Delta s_j \cdot \Phi_1^2(s_{j,m})} \quad \text{(Eq. 32-26)}$$

where, setting $s_{j+1} - s_j = \Delta s_j$ and $0,5 \cdot (s_{j+1} + s_j) = s_{j,m}$:

- Δs_j is the part number “j” (with $j = 1, 2, \ldots N$) of the structure or the structural element
- $l = \sum_{j=1}^{N} \Delta s_j$ is the height or span of the structure or the structural element
- $m(s_{j,m})$ is the mean value of the mass per unit length within the interval Δs_j
- $\Phi_1^2(s_{j,m})$ is the square of the mean value of the mode shape within the interval Δs_j.

32.5 Logarithmic decrement of damping

The logarithmic decrement of damping ä for fundamental bending mode may be estimated by expression:

$$\delta = \delta_a + \delta_s + \delta_d \quad \text{(Eq. 32-27)}$$

where:

- δ_a is the logarithmic decrement of aerodynamic damping for the fundamental mode
- δ_s is the logarithmic decrement of structural damping
- δ_d is the logarithmic decrement of damping due to special devices (tuned mass dampers, sloshing tanks etc.).

In most cases the logarithmic decrement of aerodynamic damping δ_a, for the fundamental bending mode of alongwind vibrations may be estimated by expression 32-28:

Structural type		Structural damping δ_s
reinforced concrete buildings		0,10
steel buildings		0,05
mixed structures concrete + steel		0,08
reinforced concrete towers and chimneys		0,03
unlined welded steel stacks without external thermal insulation		0,012
unlined welded steel stack with external thermal insulation		0,020
steel stack with one liner with external thermal insulation[a]	$h/b < 18$	0,020
	$20 \leq h/b < 24$	0,040
	$h/b \geq 26$	0,014
steel stack with two or more liners with external thermal insulation[a]	$h/b < 18$	0,020
	$20 \leq h/b < 24$	0,040
	$h/b \geq 26$	0,025
steel stack with internal brick liner		0,070
steel stack with internal gunite		0,030
coupled stacks without liner		0,015
guyed steel stack without liner		0,04
steel bridges + lattice steel towers	welded	0,02
	high resistance bolts	0,03
	ordinary bolts	0,05
composite bridges		0,04
concrete bridges	prestressed (no cracks)	0,04
	with cracks	0,10
Timber bridges		0,06÷0,12
Bridges, aluminium alloys		0,02
Bridges, glass or fibre reinforced plastic		0,04÷0,08
cables	parallel cables	0,006
	spiral cables	0,020

Note: The values for timber and plastic composites are indicative only. In cases where aerodynamic effects are found to be significant in the design, more refined figures are needed through specialist advice (agreed if appropriate with the competent Authority).

Note 1: For cable supported bridges the values given in Table F.2 need to be factored by 0,75.

Table 32.5 From Table F.2 - Approximate values of logarithmic decrement of structural damping in the fundamental mode, δ_s.

(a). For intermediate values of h/b, linear interpolation may be used.

$$\delta_a = \frac{c_f \cdot \rho \cdot v_m(z_s)}{2n_1 \cdot \mu_e} \approx \frac{c_f \cdot \rho \cdot b \cdot v_m(z_s)}{2n_1 \cdot m_e} \qquad \text{(Eq. 32-28)}$$

where:

- c_f is the force coefficient for wind action in the wind direction stated in Section 7
- ρ is the air density (see Sec. 4.5(1))
- b is the width of the structure (as defined in Figure 6.1)
- $v_m(z_s)$ is the mean wind velocity for $z = z_s$ (see Sec. 4.3.1 (1))
- n_1 is the fundamental frequency of along wind vibration of the structure
- m_e is the equivalent mass per unit length of the fundamental mode.

Approximate values of logarithmic decrement of structural damping, δ_s, are given in Table F.2 above.

⚠ If special dissipative devices are added to the structure, δ_d should be calculated by suitable theoretical or experimental techniques.

32.6 Verification tests

EN1991-1-4_(G).XLS. 6.22 MB. Created: 30 April 2013. Last/Rel.-date: 30 April 2013. Sheets:

— Splash

— Annex F.

EXAMPLE 32-G- Dynamic characteristics of structures: fundamental frequency - **test1**

Given: Find the fundamental flexural frequency n_1 of a cantilever beam with one mass at the end for a maximum displacement due to self weight applied in the vibration direction equal to $x_1 = 5\ \text{mm}$.

[*Reference sheet:* Annex F]-[*Cell-Range: A1:N1-A16:N16*].

Solution: From Expression (F.1):

$$n_1 = \frac{1}{2\pi}\sqrt{\frac{g}{x_1}} = \frac{1}{2\pi}\sqrt{\frac{9,81}{(5/1000)}} = 7,05\ \text{Hz}$$

with $x_1 = 5\ \text{mm} = 0,005\ \text{m}$.

example-end

EXAMPLE 32-H- Dynamic characteristics of structures: fundamental frequency - **test1b**

Given: Find the fundamental flexural frequency n_1 of a multi-storey building with an height $h = 60\ \text{m}$.

[*Reference sheet:* Annex F]-[*Cell-Range: A19:N19-A24:N24*].

Solution: From Expression (F.2): $n_1\ [\text{Hz}] = 46/h = 46/60 = 0,77\ \text{Hz}$.

example-end

EXAMPLE 32-I- Dynamic characteristics of structures: fundamental frequency - **test1c**

Given: Find the fundamental flexural frequency n_1 of a masonry chimney whose height is equal to $h = 50\ \text{m}$ above ground. The chimney, with a truncated cone shape, has an outer diameter ranging from 3,90 meters to 1,90 meters from the base to the top. The total weight of the chimney is $W_t = 534\ \text{tons}$.

[*Reference sheet:* Annex F]-[*Cell-Range: A29:N29-A64:N64*].

Solution: From Figure F.1, assuming $h_2 = h = 50\ \text{m}$ and then $h_1 = 0$ we get:

$h_{eff} = h_1 + h_2/3 = 0 + 50/3 = 16,67\ \text{m}$.

Assuming $\varepsilon_1 = 700\ [-]$ (masonry chimney) and a weight of structural part, that contributes to the stiffness of the structure, equal to the whole weight of the chimney ($W_s = W_t$), from Expression (F.3) we get:

$$n_1\ [\text{Hz}] = \frac{\varepsilon_1 \cdot b}{h_{eff}^2} \cdot \sqrt{\frac{W_s}{W_t}} = \frac{700 \cdot 1,90}{(16,67)^2} \cdot \sqrt{1} = 4,79\ \text{Hz}$$

having considered $b = 1,90\ \text{m}$ the top outer diameter of the chimney.

example-end

EXAMPLE 32-J- Dynamic characteristics of structures: fundamental frequency - **test1d**

Given: Find the fundamental ovalling frequency $n_{1,0}$ of a long cylindrical steel shell (without stiffening rings) with a diameter $b = 1,00\ \text{m}$ and a tickness $t = 3\ \text{mm}$. Assume for the shell a mass per unit area equal to $\mu_s = 22,50\ \text{kg/m}^2$.

– total sum of the lengths of each wall portion of box: $\Delta s = 4,10$ m

– enclosed cell area at mid-span (individual box): $A_j = 1,80$ m²

For the calculations let us assume:

– mass per unit length of the deck only (at mid-span): $m_d = 9600$ kg/m

– mass moment of inertia of individual box at mid-span: $I_{pj} = 2000$ kg · m²/m

– second moment of mass per unit length of individual box for vertical bending at midspan, including an associated effective width of deck: $I_j = 6000$ kg · m²/m

– mass per unit length of individual box only, at mid-span, without associated portion of deck: $m_j = 2650$ kg/m

– mean value of the sum of the squares (r_j^2): $\bar{r}^2 = \left(\sum_{j=1}^{n=3} r_j^2\right)/n = 13,5\ \text{m}^2$.

[*Reference sheet:* Annex F]-[*Cell-Range: A147:N147-A236:N236*].

Solution: From Expression (F.11), the second moment of mass per unit length of the full cross-section of the bridge at mid-span is:

$$I_p = \frac{m_d \cdot b^2}{12} + \sum(I_{pj} + m_j \cdot r_j^2) = \frac{m_d \cdot b^2}{12} + n \cdot \left[I_{pj} + m_j \cdot \left(\sum_{j=1}^{n=3} r_j^2\right)/n\right]$$

$$I_p = \frac{9600 \cdot (13)^2}{12} + 3 \cdot [2000 + 2650 \cdot 13,5] = 248525\ \text{kg} \cdot \text{m}^2/\text{m}.$$

From Expression (F.12), torsion constant (mean value) of individual box (at mid-span):

$$J_j = \frac{4A_j^2}{\oint \frac{ds}{t}} \approx \frac{4A_j^2}{\frac{\Delta s}{t_m}} = \frac{t_m \cdot 4A_j^2}{\Delta s} = \frac{(0,19) \cdot 4 \cdot (1,80)^2}{4,10} = 0,60\ \text{m}^4 \rightarrow \sum J_j = 3 \cdot 0,60 = 1,80\ \text{m}^4.$$

From previous example we have:

– second moment of area of the full cross-section of the bridge for vertical bending at mid-span equal to $I_b = 2,85\ \text{m}^4 = 3 \cdot (950000 \times 10^6)\ \text{mm}^4$

– fundamental vertical bending frequency $n_{1,B} = 4,59$ Hz

– mass per unit length of the full cross-section at midspan: $m = 13000$ kg/m.

Therefore, from Expressions (F.8), (F.9) and (F.10) we get:

$$P_1 = \frac{mb^2}{I_p} = \frac{(13000) \cdot (13)^2}{(248525)} = 8,84\ [-]$$

$$P_2 = \frac{\sum r_j^2 \cdot I_j}{b^2 \cdot I_p} = \frac{(3 \cdot 13,5) \cdot 6000}{(13)^2 \cdot (248525)} = 0,006\ [-]$$

$$P_3 = \frac{L^2 \cdot \sum J_j}{2K^2 \cdot b^2 \cdot I_b \cdot (1+\nu)} = \frac{(31)^2 \cdot (1,80)}{2\pi^2 \cdot (13)^2 \cdot (2,85) \cdot (1+0,2)} = 0,152\ [-].$$

Finally we obtain:

$$n_{1,T} = n_{1,B} \cdot \sqrt{P_1 \cdot (P_2 + P_3)} = 4,59 \cdot \sqrt{8,84 \cdot (0,006 + 0,152)} = 5,42\ \text{Hz}.$$

Plan aspect ratio: span/width = (31 m)/(13 m) = 2,38 < 6 [Satisfactory].

example-end

EXAMPLE 32-M- Fundamental mode shape - **test4**

Given: A masonry chimney is 50 meters high (see data from example 32-I on page 344). The height is discretized in equal parts of 1 meter. For each part, calculate the average height $s_{j,\,m}$ from the ground, the mass per unit length $m(s_{j,\,m})$ and the fundamental flexural mode $\Phi_1(s_{j,\,m})$ using Expression (F.13) with $z = s_{j,\,m}$. Find the equivalent mass m_e per unit length (see Expression (F.14)).

[*Reference sheet:* Annex F]-[*Cell-Range: A240:N240-A393:N393*].

Solution: Let us assume: $\Delta s_j = s_{j+1} - s_j = 1,00\ \text{m} = cost$. Therefore, we have: $\Delta s_j = 1\ \text{m}$ with $s_{j,\,m} = 0,5\ \text{m}$. Then $\Delta s_j + \Delta s_j = 2\ \text{m}$ with $s_{j,\,m} = 1,5\ \text{m}$; $\Delta s_j + \Delta s_j + \Delta s_j = 3\ \text{m}$ with $s_{j,\,m} = 2,5\ \text{m}$ and so on... Using Expression (F.13) with $\zeta = 2,0$ (for towers and chimneys), we have for example: $\Delta s_j + \Delta s_j + \Delta s_j + \ldots = 15\ \text{m}$, with $z = s_{j,\,m} = 14,5\ \text{m}$. Hence, we obtain:

Δs_j [m]	$\Phi_1^2(s_{j,m})$ [-]	Δs_j [m]	$\Phi_1^2(s_{j,m})$ [-]	Δs_j [m]	$\Phi_1^2(s_{j,m})$ [-]	Δs_j [m]	$\Phi_1^2(s_{j,m})$
1,00	0	1,00	0,01	1,00	0,1369	1,00	0,6889
1,00	0	1,00	0,0121	1,00	0,16	1,00	0,7396
1,00	0	1,00	0,0144	1,00	0,1764	1,00	0,81
1,00	0	1,00	0,0196	1,00	0,2025	1,00	0,8836
1,00	0,0001	1,00	0,0225	1,00	0,2304	1,00	0,9604
1,00	0,0001	1,00	0,0289	1,00	0,25		
1,00	0,0004	1,00	0,0324	1,00	0,2809		
1,00	0,0004	1,00	0,04	1,00	0,3136		
1,00	0,0009	1,00	0,0484	1,00	0,3481		
1,00	0,0016	1,00	0,0576	1,00	0,3844		
1,00	0,0016	1,00	0,0676	1,00	0,4356		
1,00	0,0025	1,00	0,0784	1,00	0,4761		
1,00	0,0036	1,00	0,09	1,00	0,5184		
1,00	0,0049	1,00	0,1024	1,00	0,5776		
1,00	0,0071	1,00	0,1225	1,00	0,6241		

Δs_j [m]	$m(s_{j,m})$ [kg/m]	Δs_j [m]	$m(s_{j,m})$ [kg/m]	Δs_j [m]	$m(s_{j,m})$ [kg/m]	Δs_j [m]	$m(s_{j,m})$
1,00	20501	1,00	13817	1,00	8247	1,00	3791
1,00	20021	1,00	13411	1,00	7915	1,00	3533
1,00	19545	1,00	13010	1,00	7588	1,00	3281
1,00	19075	1,00	12614	1,00	7266	1,00	3033
1,00	18610	1,00	12223	1,00	6950	1,00	2791
1,00	18149	1,00	11836	1,00	6638		
1,00	17694	1,00	11455	1,00	6331		
1,00	17243	1,00	11079	1,00	6029		
1,00	16797	1,00	10707	1,00	5732		
1,00	16357	1,00	10341	1,00	5439		
1,00	15921	1,00	9980	1,00	5152		
1,00	15490	1,00	9623	1,00	4870		
1,00	15064	1,00	9272	1,00	4593		
1,00	14644	1,00	8925	1,00	4320		
1,00	14228	1,00	8583	1,00	4053		

Figure 32.8 Calculated values (spreadsheet input).

$\Phi_1(s_{j,m}) = \left(\frac{z}{h}\right)^{\zeta} = \left(\frac{s_{j,m}}{h}\right)^2 = \left(\frac{14,5}{50}\right)^2 = 0,0841$ with $\Phi_1^2(s_{j,m}) = 0,0071$ and so on...(see tables above).

The values of the masses per unit length were calculated assuming a density for the masonry equal to 1900 kg per cubic meter (the thickness of the walls of the chimney varies from 1,25 meters at the base to 25 cm at the top).

From Tables above, we obtain:

$$\sum_{j=1}^{N} \Delta s_j \cdot \Phi_1^2(s_{j,m}) = 9,9675, \quad \sum_{j=1}^{N} \Delta s_j \cdot \Phi_1^2(s_{j,m}) \cdot m(s_{j,m}) = 49291,7393.$$

Therefore, we find:

$$m_e = \frac{\int_0^l m(s) \cdot \Phi_1^2(s) ds}{\int_0^l \Phi_1^2(s) ds} \approx \frac{\sum_{j=1}^{N} \Delta s_j \cdot \Phi_1^2(s_{j,m}) \cdot m(s_{j,m})}{\sum_{j=1}^{N} \Delta s_j \cdot \Phi_1^2(s_{j,m})} = \frac{49291,7393}{9,9675} \approx 4945 \text{ kg/m}.$$

Note: for cantilevered structures with a varying mass distribution m_e may be approximated by average value of $m(s)$ over the upper third of the structure. In this case: $\frac{h}{3} = \frac{50}{3} = 16,7 \text{ m}$.

Using data from tables above, for $\sum \Delta s_j \geq h_{III} = h - \frac{h}{3} = 33,3 \text{ m}$ we get ($j \geq 33$):

$$m_e \approx \frac{(7266 + 6950 + 6638 + \ldots + 3281 + 3033 + 2791)}{17} = 4929,53 \text{ kg/m}.$$

The last calculation remains a good approximation even if the chimney is of truncated conical shape.

example-end

EXAMPLE 32-N- Logarithmic decrement of damping - **test5**

Given: Find the logarithmic decrement of damping δ for fundamental bending mode of the masonry chimney analysed in the previous examples. Assume a mean wind velocity (at height $z_s = 0,6 \cdot h$) equal to $v_m(z_s) = 28 \text{ m/s}$. The stack height is 50 meters from the ground. Assume a force coefficient (see Sec. 7) c_f round to 1,05.

[*Reference sheet:* Annex F]-[*Cell-Range: A430:N430-A464:N464*].

Solution: From Figure 6.1: $z_s = 0,6 \cdot h = 0,6 \cdot 50 = 30 \text{ m}$ (case a: vertical structures). The outer diameter of the chimney at height of 30 meters above the ground is equal (say) to b = 2,70 meters. From previous example (see example 32-I on page 344), the fundamental frequency of along wind vibration of the structure is $n_1 = 4,79 \text{ Hz}$ and the equivalent mass m_e (see previous example 32-M) equal to 4945 kg/m. Therefore, the logarithmic decrement of aerodynamic damping (for the fundamental mode) is:

$$\delta_a = \frac{c_f \cdot \rho \cdot v_m(z_s)}{2n_1 \cdot \mu_e} \approx \frac{c_f \cdot \rho \cdot b \cdot v_m(z_s)}{2n_1 \cdot m_e} = \frac{1,05 \cdot (1,25) \cdot (2,70) \cdot (28)}{2 \cdot 4,79 \cdot (4945)} = 0,0021 .$$

From Table F.2 ("*Approximate values of logarithmic decrement of structural damping in the fundamental mode,* δ_s"), for reinforced concrete towers and chimneys we have: $\delta_s = 0,030$.

From Expression (F.15), finally we find:

$$\delta = \delta_a + \delta_s + \delta_d = 0,002 + 0 + 0,030 = 0,032 \; [-] ,$$

in this case having considered equal to zero the logarithmic decrement of damping due to special devices.

example-end

32.7 References [Section 32]

EN 1991-1-4:2005/A1:2010. Eurocode 1: Actions on structures - Part 1-4: General actions - Wind actions. Brussels: CEN/TC 250 - Structural Eurocodes, April 2010.

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Section 33 Eurocode 1 EN 1991-1-5 Section 5 (Page 17 to 19)

33.1 General

Thermal actions shall be classified as variable and indirect actions, see EN 1990:2002, 1.5.3 and 4.1.1. All values of thermal actions given in this Part are characteristic values unless it is stated otherwise.

> In general thermal variations cause deformations in single structural elements as well as in the overall structure itself. If the structure is hyperstatic, a further consequence of thermal variations is the emergence of coactive stress states. Therefore, the effects of thermal variations may involve aspects of a structure's functionality as well as its safety.

Characteristic values of thermal actions as given in this Part are values with an annual probability of being exceeded of 0,02, unless otherwise stated, e.g. for transient design situations.

DESIGN SITUATIONS Thermal actions shall be determined for each relevant design situation identified in accordance with EN 1990. Structures not exposed to daily and seasonal climatic and operational temperature changes may not need to be considered for thermal actions. The elements of loadbearing structures shall be checked to ensure that thermal movement will not cause overstressing of the structure, either by the provision of movement joints or by including the effects in the design.

REPRESENTATION OF ACTIONS Daily and seasonal changes in shade air temperature, solar radiation, reradiation, etc., will result in variations of the temperature distribution within individual elements of a structure. The magnitude of the thermal effects will be dependent on local climatic conditions, together with the orientation of the structure, its overall mass, finishes (e.g. cladding in buildings), and in the case of building structures, heating and ventilation regimes and thermal insulation.

33.2 Temperature changes in buildings

Thermal actions on buildings due to climatic and operational temperature changes shall be considered in the design of buildings where there is a possibility of the ultimate or serviceability limit states being exceeded due to thermal movement and/or stresses.

DETERMINATION OF TEMPERATURES Thermal actions on buildings due to climatic and operational temperature changes should be determined in accordance with the principles and rules provided in this Section taking into account national (regional) data and experience. The climatic effects shall be determined by considering the variation of shade air temperature and solar radiation. Operational effects (due to heating, technological or industrial processes) shall be considered in accordance with the particular project. The uniform temperature component of a structural element ΔT_u is defined as:

$$\Delta T_u = T - T_0 \qquad \textbf{(Eq. 33-29)}$$

where T is an average temperature of a structural element due to climatic temperatures in winter or summer season and due to operational temperatures.

DETERMINATION OF TEMPERATURE PROFILES The temperature T in Eq. 33-29 should be determined as the average temperature of a structural element in winter or summer using a temperature profile. In the case of a sandwich element T is the average temperature of a particular layer. When elements of one layer are considered and when the environmental conditions on both sides are similar, T may be approximately determined as the average of inner and outer environment temperature T_{in} and T_{out}. The temperature of the inner environment, T_{in} should be determined in accordance with Table 5.1. The temperature of the outer environment, T_{out} should be determined in accordance with:

— Table 5.2 for parts located above ground level
— Table 5.3 for underground parts.

The temperatures T_{out} for the summer season as indicated in Table 5.2 are dependent on the surface absorptivity and its orientation:

— the maximum is usually reached for surfaces facing the west, south-west or for horizontal surfaces
— the minimum (in °C about half of the maximum) for surfaces facing the north.

Season	Temperature T_{in}
Summer	$T_1 = 20°C$[(a)]
Winter	$T_2 = 25°C$[(a)]

Table 33.6 From Table 5.1 - Indicative temperatures of inner environment T_{in}.

(a). Values for T_1 and T_2 may be specified in the National Annex. When no data are available the values $T_1 = 20°C$ and $T_2 = 25°C$ are recommended.

Season	Significant factor		Temperature Tout [°C][(a)]
Summer	Relative absorptivity depending on surface colour	0,5 bright light surface	$T_{max} + T_3$
		0,7 light coloured surface	$T_{max} + T_4$
		0,9 dark surface	$T_{max} + T_5$
Winter			T_{min}

Table 33.7 From Table 5.2 - Indicative temperatures T_{out} for buildings above the ground level.

(a). Values of the maximum shade air temperature T_{max}, minimum shade air temperature T_{min}, and solar radiation effects T_3, T_4, and T_5 may be specified in the National Annex. If no data are available for regions between latitudes 45°N and 55°N the values $T_3 = 0°C$, $T_4 = 2°C$, and $T_5 = 4°C$ are recommended, for North-East facing elements and $T_3 = 18°C$, $T_4 = 30°C$ and $T_5 = 42°C$ for South-West or horizontal facing elements.

Season	Depth below the ground level	Temperature T_{out} [°C][(a)]
Summer	Less than 1 m	T_6
	More then 1 m	T_7
Winter	Less than 1 m	T_8
	More then 1 m	T_9

Table 33.8 From Table 5.3 - Indicative temperatures T_{out} for underground parts of buildings.

(a). Values T_6, T_7, T_8 and T_9 may be specified in the National Annex. If no data are available for regions between latitudes 45°N and 55°N the values $T_6 = 8°C$, $T_7 = 5°C$, $T_8 = -5°C$ and $T_9 = -3°C$ are recommended.

33.3 Verification tests

EN1991-1-5_(A).XLS. 8.31 MB. Created: 15 November 2013. Last/Rel.-date: 15 November 2013. Sheets:

- Splash
- CodeSec5.

EXAMPLE 33-O- Temperature changes in buildings - Determination of temperature profiles - **test1**

Given: Let us analyse a regular steel framework that forms three 5,0 m bays (with an overall plane surface of 15,0 m) and two floors, 3,0 m in height (for a total of 6,0 m), as represented in Figure 33.9 (see below).

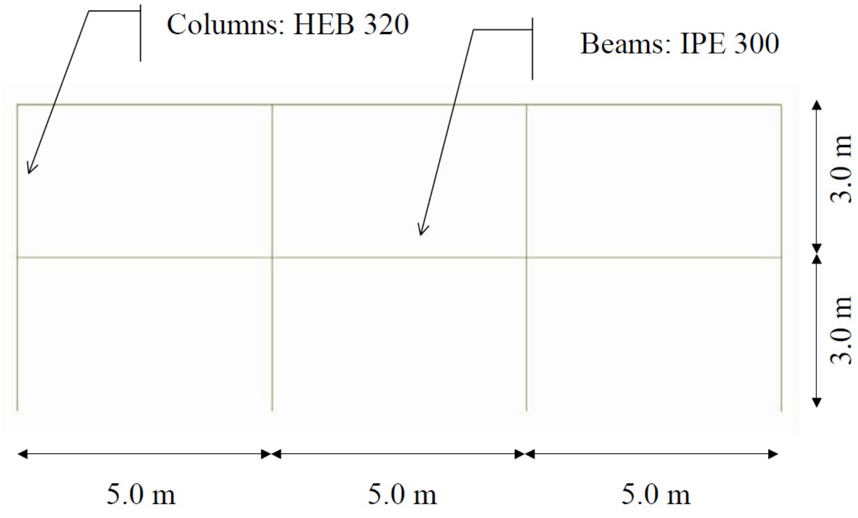

Figure 33.9 Example of a steel frame. See "*Development of skills facilitating implementation of Eurocodes - Handbook 3 - Action effects for buildings*". Leonardo da Vinci Pilot Project CZ/02/B/F/PP-134007. 10/2005

The following structural elements go to make up the framework:

— beams (spanning 5,0 m): IPE 300

— columns: HEB 320.

Let us consider four different uniform temperature components:

1. **Case 1**: heating of every structural element (beams and columns) of the structure (summer season)
2. **Case 2**: cooling of every structural element (beams and columns) of the structure (winter season)
3. **Case 3**: heating of the external beams and columns
4. **Case 4**: cooling of the external beam and columns.

Find the four thermal (characteristic) load cases upon the steel frame. Assumptions:

a. South-West facing elements

b. light coloured surfaces.

[*Reference sheet:* CodeSec5]-[*Cell-Range: A1:O1-A260:O260*].

Solution: From Table 5.1 - "*Indicative temperatures of inner environment T_{in}*" we have $T_1 = 20°C$ (Summer) and $T_2 = 25°C$ (Winter).

Let us assume (say):

– maximum shade air temperature: $T_{max} = 40°C$

– minimum shade air temperature: $T_{min} = -9°C$.

From Table 5.2 - "*Indicative temperatures Tout for buildings above the ground level*" for light coloured surfaces we have $T_{out} = T_{max} + T_4 = (40 + 30) = 70°C$ (Summer) and $T_{out} = T_{min} = -9°C$ (Winter).

Average temperature of a structural element:(1)

$T = (T_{in} + T_{out})/2 = (20 + 70)/2 = 45°C$ (Summer),

$T = (T_{in} + T_{out})/2 = (25 - 9)/2 = 8°C$ (Winter).

Assuming an initial temperature $T_0 = 10°C$ (see Annex A - Sec. A.1(3)), the uniform temperature component of a structural element is (mean value):(2)

$\Delta T_u = T - T_0 = (45 - 10) = 35°C$ (Summer), $\Delta T_u = T - T_0 = (8 - 10) = -2°C$ (Winter).

Therefore, above ground level ($\Delta h_g = 0$), we get:

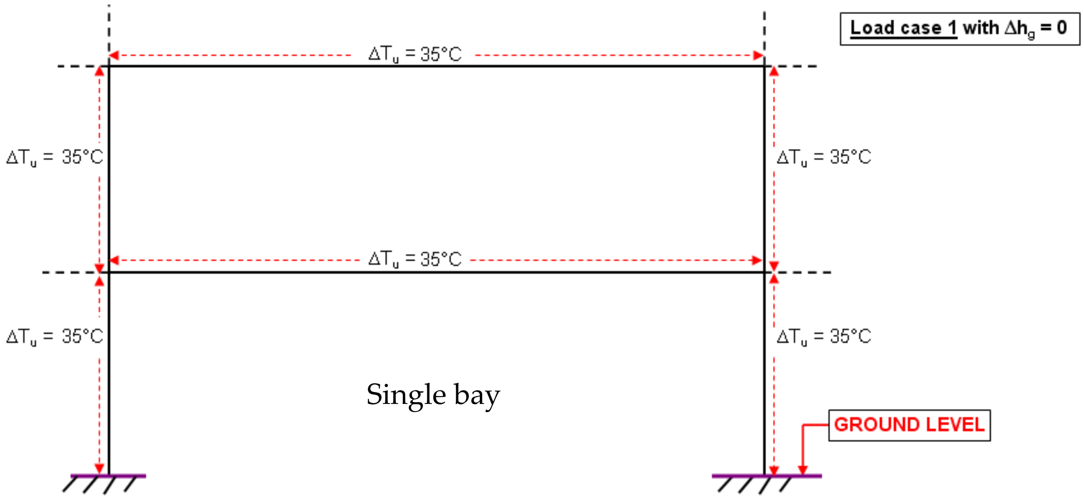

Figure 33.10 Heating of every structural element (beams and columns) of the structure (summer season).

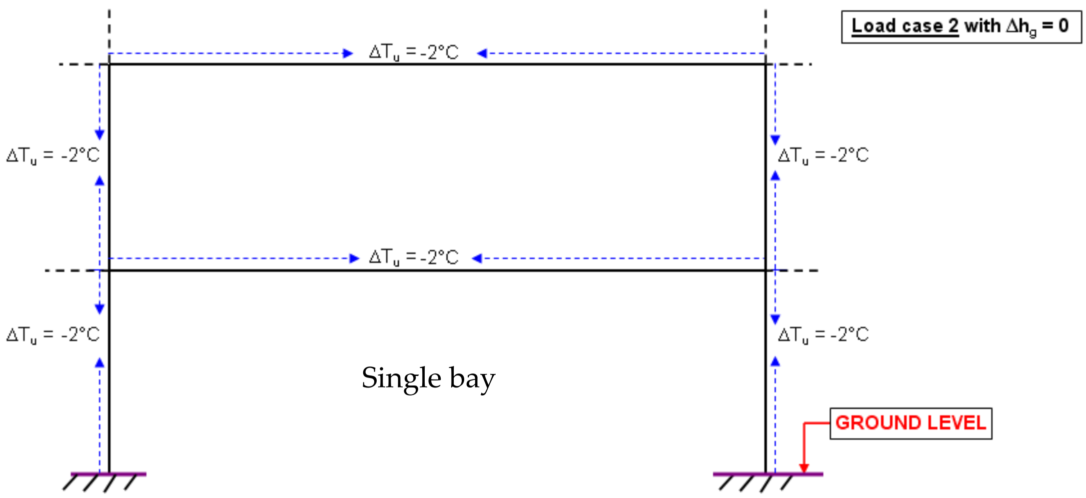

Figure 33.11 Cooling of every structural element (beams and columns) of the structure (winter season).

(1) See EN 1991-1-5, Section 5.3(1) - NOTE 2.

(2) See EN 1991-1-5, Section5.2(5) - Eq. (5.1).

$T = (T_{in} + T_{out})/2 = (25 - 5)/2 = 10°C$ (Winter with $T_8 = -5°C$).

Assuming an initial temperature $T_0 = 10°C$ (see Annex A - Sec. A.1(3)), the uniform temperature component of a structural element is (mean value):(1)

$\Delta T_u = T - T_0 = (14 - 10) = 4°C$ (Summer), $\Delta T_u = T - T_0 = (10 - 10) = 0°C$ (Winter).

Zone with $h > 1$ m **(zone A):**

Average temperature of a structural element:(2)

$T = (T_{in} + T_{out})/2 = (20 + 5)/2 = 12,5°C$ (Summer with $T_7 = 5°C$),

$T = (T_{in} + T_{out})/2 = (25 - 3)/2 = 11°C$ (Winter with $T_9 = -3°C$).

Assuming an initial temperature $T_0 = 10°C$ (see Annex A - Sec. A.1(3)), the uniform temperature component of a structural element is (mean value):(3)

$\Delta T_u = T - T_0 = (12,5 - 10) = 2,5°C$ (Summer), $\Delta T_u = T - T_0 = (11 - 10) = 1°C$ (Winter).

Therefore, above ground level ($\Delta h_g = 0$), we get:

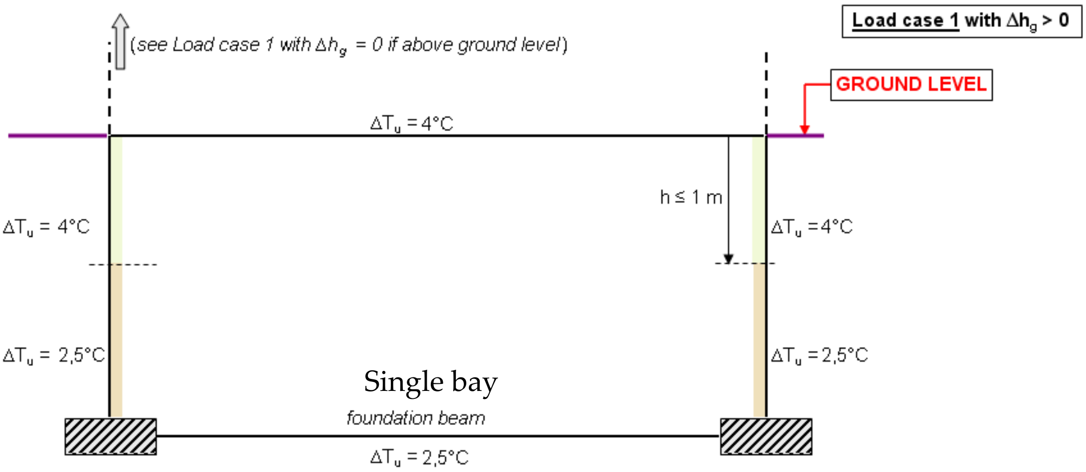

Figure 33.14 Heating of every structural element (beams and columns) of the structure (summer season).

(1) See EN 1991-1-5, Section 5.3(1) - NOTE 2.
(1) See EN 1991-1-5, Section5.2(5) - Eq. (5.1).
(2) See note 1.
(3) See note 2.

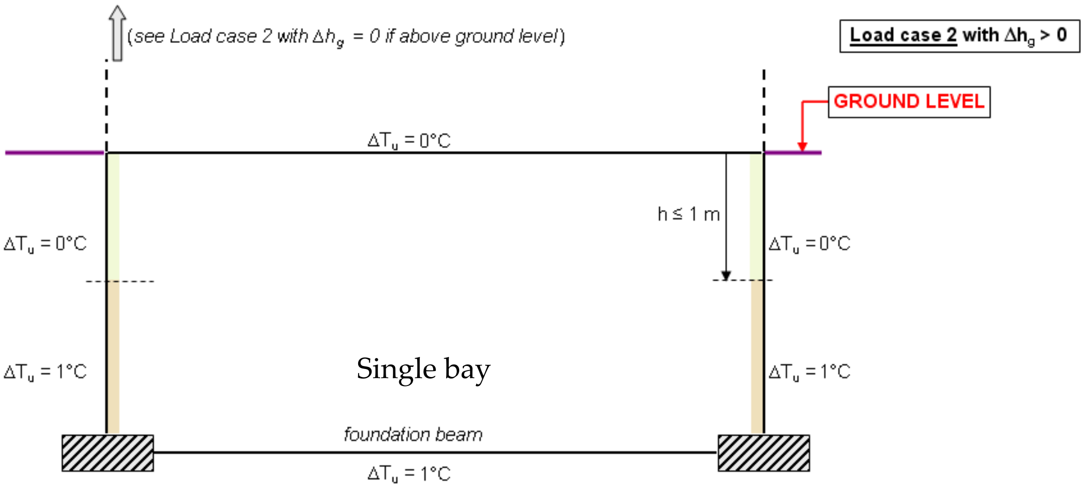

Figure 33.15 Cooling of every structural element (beams and columns) of the structure (winter season).

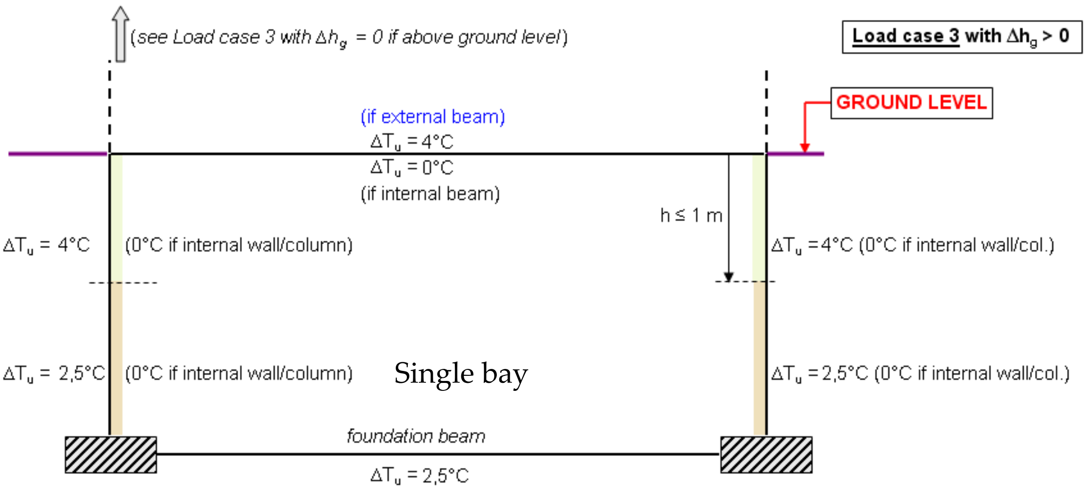

Figure 33.16 Heating of the external beams and columns.

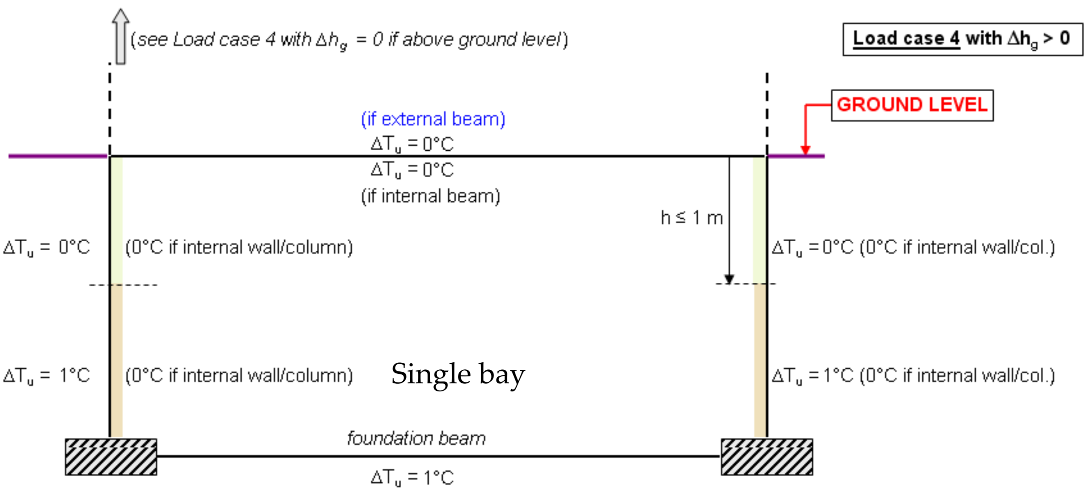

Figure 33.17 Cooling of the external beam and columns.

33.4 References [Section 33]

EN 1991-1-5:2003. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, November 2003 (DAV).

EN 1991-1-5:2003/AC:2009. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2009.

Manual for the design of building structures to Eurocode 1 and Basis of Structural Design April 2010. © 2010 The Institution of Structural Engineers.

Development of skills facilitating implementation of Eurocodes - Handbook 3 - Action effects for buildings. Leonardo da Vinci Pilot Project CZ/02/B/F/PP-134007. Aachen 10/2005.

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Section 34 **Eurocode 1**
EN 1991-1-5
Section 6

34.1 Temperature changes in bridges

34.1.1 Bridge decks

Three types of bridge superstructures are distinguished in EN 1991-1-5. For the purposes of this Part, bridge decks are grouped as follow:

- Type 1. Steel deck:
  - — steel box girder
  - — steel truss or plate girder
- Type 2. Composite deck
- Type 3. Concrete deck:
  - — concrete slab
  - — concrete beam
  - — concrete box girder.

THERMAL ACTIONS Representative values of thermal actions should be assessed by the uniform temperature component (see EN 1991-1-5, Sec. 6.1.3) and the temperature difference components (see EN 1991-1-5, Sec. 6.1.4).

The vertical temperature difference component should generally include the non-linear component. Either Approach 1 or Approach 2 should be used.

34.1.2 Thermal actions

UNIFORM TEMPERATURE COMPONENT The uniform temperature component depends on the minimum and maximum temperature which a bridge will achieve. This results in a range of uniform temperature changes which, in an unrestrained structure would result in a change in element length.

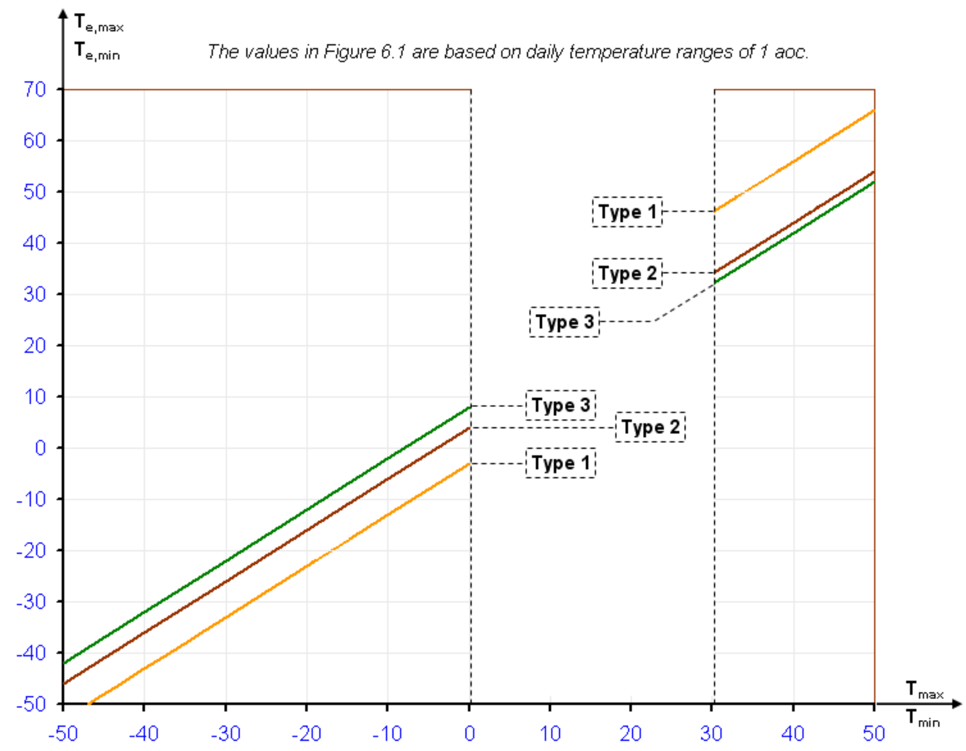


Figure 34.18 From Figure 6.1 - Correlation between minimum (maximum) shade air temperature T_{min} (T_{max}) and minimum (maximum) uniform bridge temperature component $T_{e,min}$ ($T_{e,max}$).

The minimum and maximum uniform (effective) bridge temperatures $T_{e,min}$ ($T_{e,max}$) can be determined from the relationship given in Fig. 6.1 on the basis of isotherms of shade air temperatures T_{min} (T_{max}). The characteristic values of minimum and maximum shade air temperatures for a site location may be obtained e.g. from national maps of isotherms. These characteristic values represent shade air temperatures at mean sea level in open country being exceeded by annual extremes with the probability of 0,02. The relationship given in Fig. 6.1 is based on a daily temperature range of 10°C. Such a range may be considered as appropriate for most Member States. The maximum uniform temperature component $T_{e,max}$ and the minimum uniform temperature component $T_{e,min}$ for the three types of bridge decks may be determined from the following relationships based on Figure 6.1:

$$\begin{cases} T_{e,\,max} = T_{max} + 16°C \\ T_{e,\,max} = T_{max} + 4°C \quad \text{for } 30°C \le T_{max} \le 50°C\,. \\ T_{e,\,max} = T_{max} + 2°C \end{cases} \qquad \textbf{(Eq. 34-30)}$$

$$\begin{cases} T_{e,\,min} = T_{min} - 3°C \\ T_{e,\,min} = T_{min} + 4°C \quad \text{for } -50°C \le T_{max} \le 0°C\,. \\ T_{e,\,min} = T_{min} + 8°C \end{cases} \qquad \textbf{(Eq. 34-31)}$$

For steel truss and plate girders the maximum values given for Type 1 may be reduced by 3°C.

For construction works located in specific climatic regions as in e.g. frost pockets, additional information should be obtained and evaluated.

Minimum shade air temperature (T_{min}) and maximum shade air temperature (T_{max}) for the site shall be derived from isotherms in accordance with 6.1.3.2. The National Annex may specify $T_{e,min}$ and $T_{e,max}$. Figure 6.1 below gives recommended values.

SHADE AIR TEMPERATURE Characteristic values of minimum and maximum shade air temperatures for the site location shall be obtained, e.g. from national maps of isotherms. Information (e.g. maps of isotherms) on minimum and maximum shade air temperatures to be used in a Country may be found in its National Annex. Where an annual probability of being exceeded of 0,02 is deemed inappropriate, the minimum shade air temperatures and the maximum shade air temperatures should be modified in accordance with annex A.

RANGE OF UNIFORM BRIDGE TEMPERATURE COMPONENT The values of minimum and maximum uniform bridge temperature components for restraining forces shall be derived from the minimum (T_{min}) and maximum (T_{max}) shade air temperatures (see 6.1.3.1 (3) and 6.1.3.1 (4)). The initial bridge temperature T_0 at the time that the structure is restrained may be taken from annex A for calculating contraction down to the minimum uniform bridge temperature component and expansion up to the maximum uniform bridge temperature component. Thus the characteristic value of the maximum contraction range of the uniform bridge temperature component, $\Delta T_{N,\,con}$ should be taken as:

$$\Delta T_{N,\,con} = T_0 - T_{e,\,min} \qquad \text{(Eq. 34-32)}$$

and the characteristic value of the maximum expansion range of the uniform bridge temperature component, $\Delta T_{N,\,exp}$ should be taken as:

$$\Delta T_{N,\,exp} = T_{e,\,max} - T_0 \,. \qquad \text{(Eq. 34-33)}$$

34.2 Temperature difference components

34.2.1 Vertical linear component (Approach 1)

For the vertical temperature difference component, two alternative approaches are provided in EN 1991-1-5 which may be nationally selected: (1) linear, or (2) non linear temperature distribution.

The models applied in the linear approach are given in Table 6.1 ("*Recommended values of linear temperature difference component for different type of bridge decks for road, foot and railway bridges*") for bridges based on a depth of surfacing of 50 mm. For other surfacing thicknesses, the coefficient k_{sur} should be applied (see Table 6.2 - "*Recommended values of ksur to account for different surfacing thickness*").

Type of Deck[(a)]	Top warmer then bottom $\Delta T_{M,heat}$ [°C]	Bottom warmer than top $\Delta T_{M,cool}$ [°C]
Type 1. Steel deck	18	13
Type 2. Composite deck	15	18
Type 3. Concrete deck:		
- concrete box girder	10	5
- concrete beam	15	8
- concrete slab	15	8

Table 34.9 From Table 6.1 - Recommended values of linear temperature difference component for different type of bridge decks for road, foot and railway bridges.

(a). The values given in the table represent upper bound values of the linearly varying temperature difference component for representative sample of bridge geometries. The values given in the table are based on a depth of surfacing of 50 mm for road and railway bridges. For other depths of surfacing these values should be multiplied by the factor ksur. Recommended values for the factor ksur is given in Table 6.2.

Road, foot and railway bridges						
Surface Thickness	Type 1		Type 2		Type 3	
	Top warmer then bottom	Bottom warmer then top	Top warmer then bottom	Bottom warmer then top	Top warmer then bottom	Bottom warmer then top
[mm]	k_{sur}	k_{sur}	k_{sur}	k_{sur}	k_{sur}	k_{sur}
unsurfaced	0,7	0,9	0,9	1,0	0,8	1,1
water-proofed[(a)]	1,6	0,6	1,1	0,9	1,5	1,0
50	1,0	1,0	1,0	1,0	1,0	1,0
100	0,7	1,2	1,0	1,0	0,7	1,0
150	0,7	1,2	1,0	1,0	0,5	1,0
ballast (750 mm)	0,6	1,4	0,8	1,2	0,6	1,0

Table 34.10 From Table 6.2 - Recommended values of k_{sur} to account for different surfacing thickness.

(a). These values represent upper bound values for dark colour.

34.2.2 Vertical temperature components with non-linear effects (Approach 2)

Values of vertical temperature differences for bridge decks to be used in a Country may be found in its National Annex.

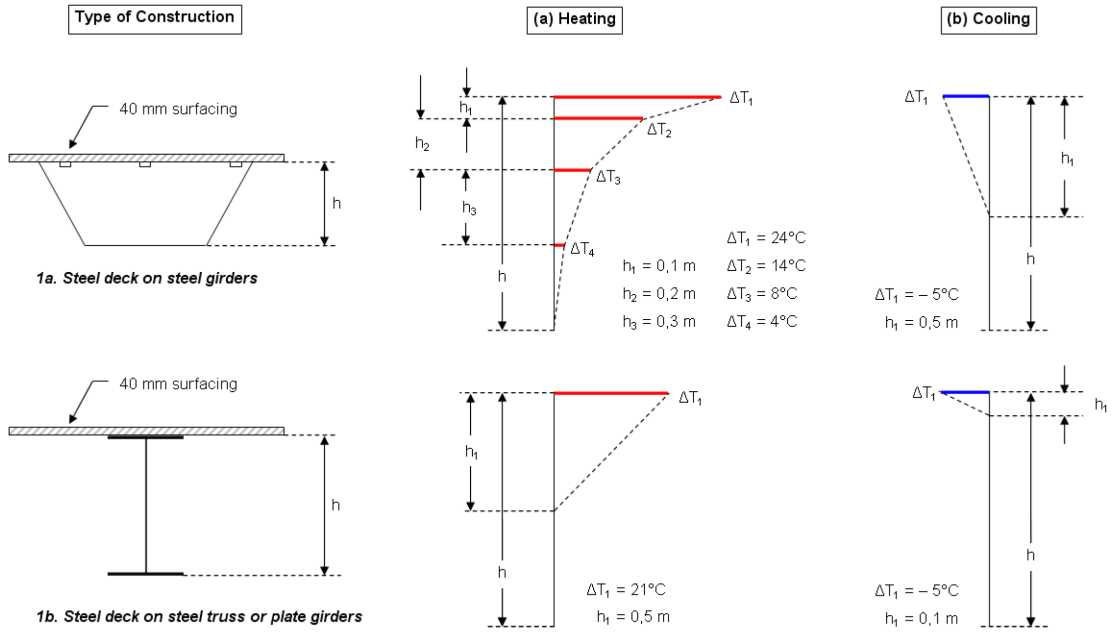


Figure 34.20 From Figure 6.2a - Temperature differences for bridge decks - Type 1: Steel Decks.

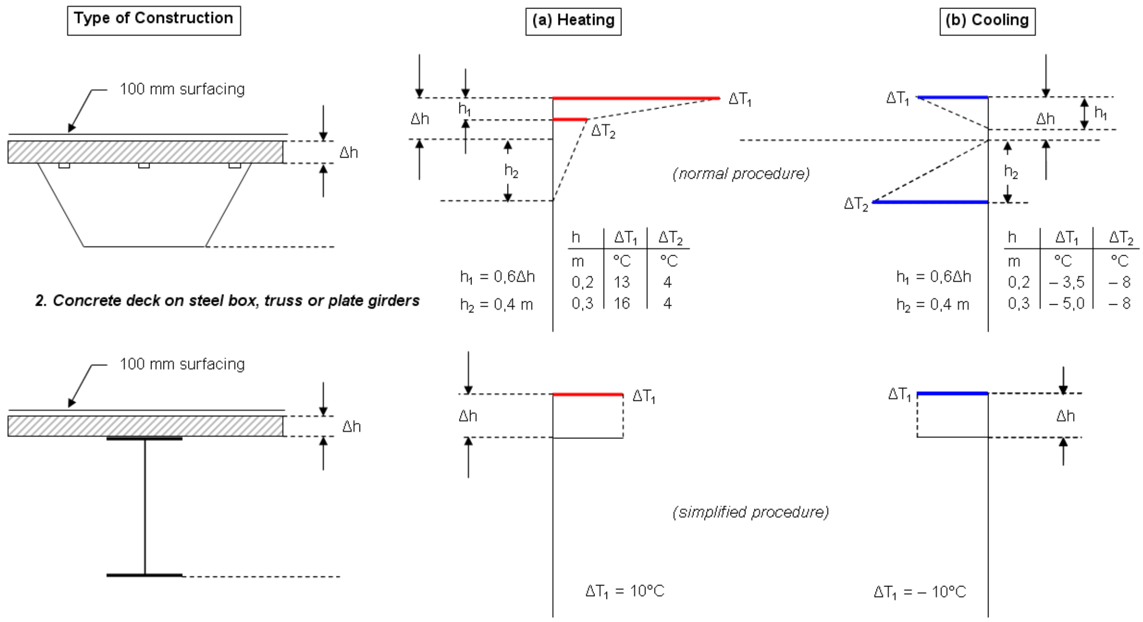
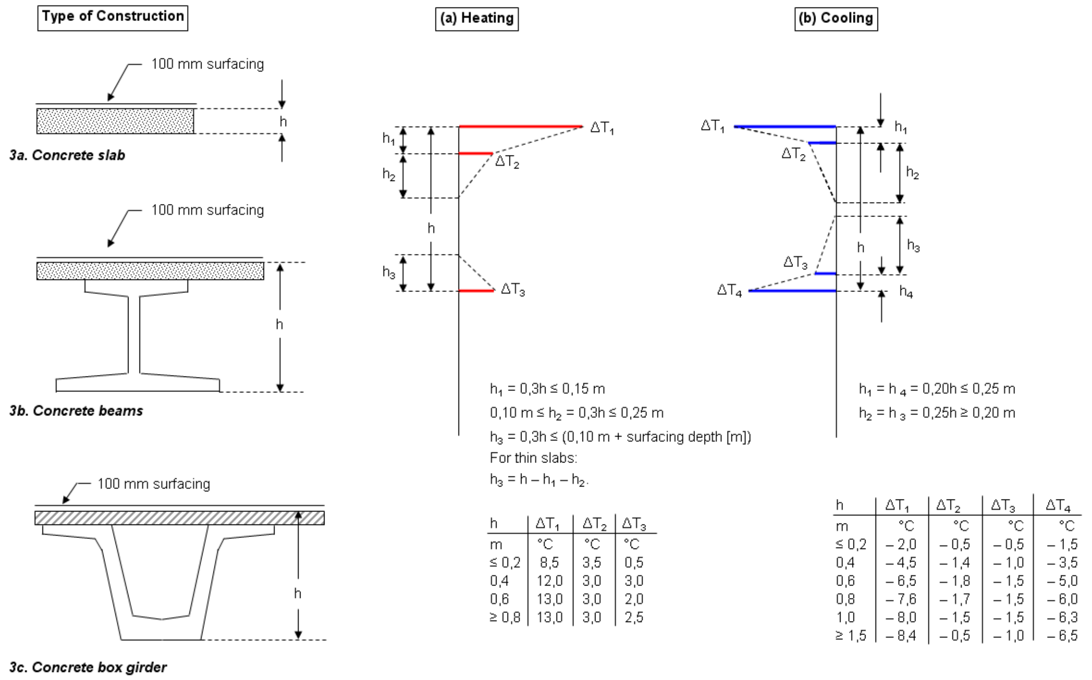


Figure 34.19 From Figure 6.2b - Temperature differences for bridge decks - Type 2: Composite Decks.



h	ΔT_1	ΔT_2	ΔT_3
m	°C	°C	°C
≤ 0,2	8,5	3,5	0,5
0,4	12,0	3,0	3,0
0,6	13,0	3,0	2,0
≥ 0,8	13,0	3,0	2,5

h	ΔT_1	ΔT_2	ΔT_3	ΔT_4
m	°C	°C	°C	°C
≤ 0,2	– 2,0	– 0,5	– 0,5	– 1,5
0,4	– 4,5	– 1,4	– 1,0	– 3,5
0,6	– 6,5	– 1,8	– 1,5	– 5,0
0,8	– 7,6	– 1,7	– 1,5	– 6,0
1,0	– 8,0	– 1,5	– 1,5	– 6,3
≥ 1,5	– 8,4	– 0,5	– 1,0	– 6,5

Figure 34.21 From Figure 6.2c - Temperature differences for bridge decks - Type 3: Concrete Decks.

Recommended values are given in Figures 6.2a/6.2b/6.2c and are valid for 40 mm surfacing depths for deck type 1 and 100 mm for deck types 2 and 3. For other depths of surfacing see Annex B. In these figures “heating” refers to conditions such that solar radiation and other effects cause a gain in heat through the top surface of the bridge deck. Conversely, “cooling” refers to conditions such that heat is lost from the top surface of the bridge deck as a result of re-radiation and other effects.

34.2.3 Simultaneity of uniform and temperature difference components

In some cases, it may be necessary to take into account both the temperature difference $\Delta T_{M,\,heat}$ (or $\Delta T_{M,\,cool}$) and the maximum range of uniform bridge temperature component $\Delta T_{N,\,exp}$ (or $\Delta T_{N,\,con}$) given as:

$$\begin{cases} \Delta T_{M,\,heat} + \omega_N \cdot \Delta T_{N,\,exp} \\ \Delta T_{M,\,cool} + \omega_N \cdot \Delta T_{N,\,con} \end{cases} \qquad \textbf{(Eq. 34-34)}$$

$$\begin{cases} \omega_M \cdot \Delta T_{M,\,heat} + \Delta T_{N,\,exp} \\ \omega_M \cdot \Delta T_{M,\,cool} + \Delta T_{N,\,con} \end{cases} \qquad \textbf{(Eq. 34-35)}$$

where the most adverse effect should be chosen. The National annex may specify numerical values of ω_N and ω_M. If no other information is available, the recommended values (reduction factors) for ω_N and ω_M are: $\omega_N = 0,35$, $\omega_M = 0,35$.

Where both linear and non-linear vertical temperature differences are used (see 6.1.4.2) ΔT_M should be replaced by ΔT which includes ΔT_M and ΔT_E (see Figures 6.2a/6.2b and 6.2c), where:

- ΔT_M linear temperature difference component
- ΔT_E non-linear part of the difference component
- ΔT sum of linear temperature difference component and non-linear part of the temperature difference component.

34.2.4 Bridge Piers: temperature differences

For concrete piers (hollow or solid), the linear temperature differences between opposite outer faces should be taken into account. The National annex may specify values for linear temperature differences. In the absence of detailed information the recommended value is 5°C.

For walls the linear temperature differences between the inner and outer faces should be taken into account. The National annex may specify values for linear temperature differences. In the absence of detailed information the recommended value is 15°C.

34.3 Verification tests

EN1991-1-5_(A)_2.XLS. 8.31 MB. Created: 20 November 2013. Last/Rel.-date: 20 November 2013. Sheets:

— Splash

— CodeSec6.

EXAMPLE 34-Q- Characteristic thermal actions in bridges - Consideration of thermal actions - **test1**

Given: Determine the maximum uniform temperature component $T_{e,\,max}$ and the minimum uniform temperature component $T_{e,\,min}$ for the three types of bridge decks determined from the relationships based on Figure 6.1. Let us assume that the characteristic values of minimum T_{min} and maximum T_{max} shade air temperatures for a site location (say the city of Birmingham) was obtained e.g. from the UK national maps of isotherms. These characteristic values represent shade air temperatures at mean sea level in open country being exceeded by annual extremes with the probability of 0,02.

[*Reference sheet:* CodeSec6]-[*Cell-Range: A1:O1-A86:O86*].

Solution: From the UK isotherms maps (see "*Manual for the design of building structures to Eurocode 1 and Basis of Structural Design*" - The Institution of Structural Engineers Manual for the design of building structures to Eurocode 1. April 2010), we have (near Birmingham):

$T_{max} = 34°C$ (rounded value for probability p = 0,02)

$T_{min} = -18°C$ (rounded value for probability p = 0,02).

Therefore, we get:

$$\begin{cases} T_{e,\,max} = T_{max} + 16°C = (34 + 16) = 50°C \\ T_{e,\,max} = T_{max} + 4°C = (34 + 4) = 38°C \\ T_{e,\,max} = T_{max} + 2°C = (34 + 2) = 36°C \end{cases} \quad \text{for } 30°C \le T_{max} \le 50°C\,.$$

$$\begin{cases} T_{e,\,min} = T_{min} - 3°C = (-18 - 3) = -21°C \\ T_{e,\,min} = T_{min} + 4°C = (-18 + 4) = -14°C \\ T_{e,\,min} = T_{min} + 8°C = (-18 + 8) = -10°C \end{cases} \quad \text{for } -50°C \le T_{max} \le 0°C\,.$$

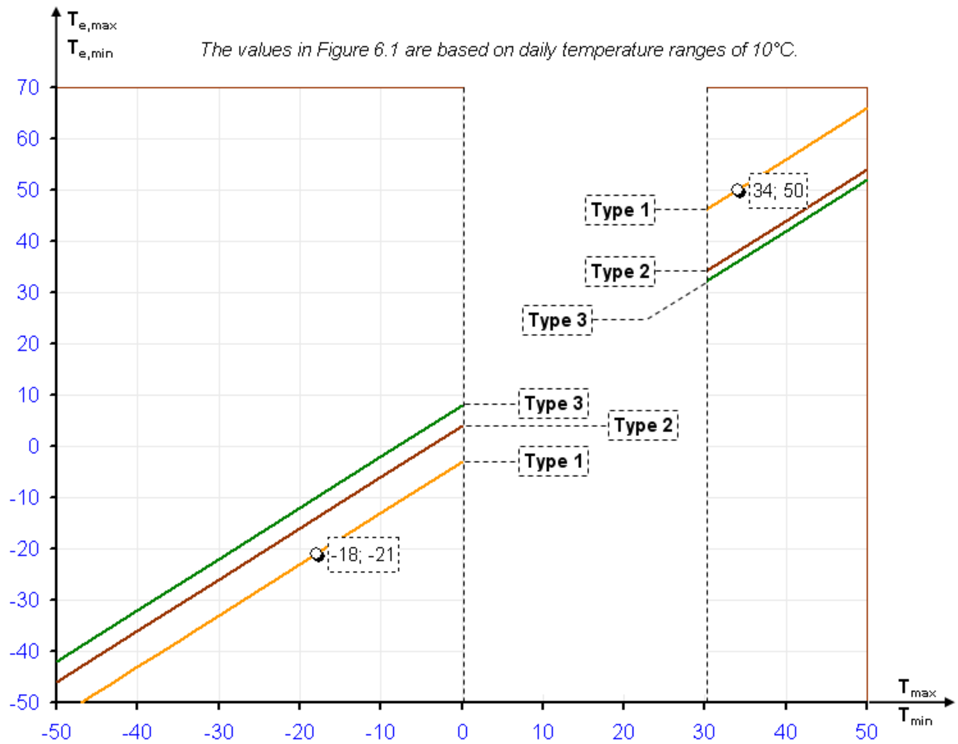


Figure 34.22 Excel® output graph (for Bridge deck Type 1).

The algorithm to draw the graph above is the same. We omit the other two cases (Type 2 and Type 3).

example-end

EXAMPLE 34-S- Characteristic thermal actions in bridges - Temperature difference components - **test3**

Given: Assuming the same assumptions form the previous examples, find the vertical linear temperature component (Approach 1) for a bridge deck Type 1 with a surface thickness equal to 100 mm.

[*Reference sheet:* CodeSec6]-[*Cell-Range: A173:O173-A238:O238*].

Solution: Entering Table 6.1 - "*Recommended values of linear temperature difference component for different types of bridge decks for road, foot and railway bridges*" with steel deck Type 1 we get (for $k_{sur} = 1$):

– linear temperature difference component (heating): $\Delta T_{M,\,heat} = 18°C$;

– linear temperature difference component (cooling): $\Delta T_{M,\,cool} = 13°C$.

The values given above represent upper bound values of the linearly varying temperature difference component for representative sample of bridge geometries. The values given in Table 6.1 are based on a depth of surfacing of 50 mm for road and railway bridges.

For other depths of surfacing these values should be multiplied by the factor k_{sur}. Recommended values for the factor k_{sur} are given in Table 6.2. For surface thickness equal to 100 mm and for bridge deck Type 1 we have:

$$k_{sur} = \begin{cases} 0,7 \ \textit{(top warmer than bottom)} \\ 1,2 \ \textit{(bottom warmer then top)}. \end{cases}$$

Hence we get (for surface thickness equal to 100 mm):

$$\Delta T_{M,\,heat} = k_{sur} \cdot (18°C) = 0,7 \cdot (18) = 12,6°C$$

$$\Delta T_{M,\,cool} = k_{sur} \cdot (13°C) = 1,2 \cdot (13) = 15,6°C\,.$$

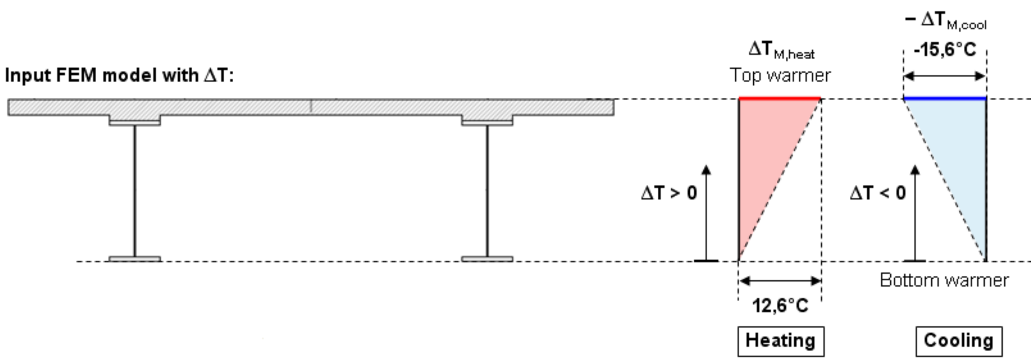


Figure 34.23 Excel® output graph (for Bridge deck Type 1): characteristic values.

EXAMPLE 34-T- Characteristic thermal actions in bridges - Vertical temperature (Approach 2) - **test3b**

Given: Let us consider a bridge deck Type 3 (prestressed precast concrete beam bridge). The height of the precast beam is 36 in = 0.91 m (rounded value). The thickness of the reinforced concrete bridge deck is 25 cm.

Assuming a surfacing depth equal to 100 mm find the temperature difference for heating and cooling (see Figure 6.2c).

[*Reference sheet:* CodeSec6]-[*Cell-Range: A242:O242-A365:O365*].

Solution: Entering Table 6.2c with $h = (0,91 + 0,25) = 1,16\ \text{m}$ we get:

(a) Heating

$h_1 = 0,3 \cdot h = 0,3 \cdot (1,16) = 0,35\ \text{m} \le 0,15\ \text{m} \quad \rightarrow \quad h_1 = 0,15\ \text{m}$

$h_2 = 0,3 \cdot h = 0,3 \cdot (1,16) = 0,35\ \text{m}$ with $0,10\ \text{m} \le h_2 \le 0,25\ \text{m} \quad \rightarrow \quad h_2 = 0,25\ \text{m}$

$h_2 = 0,3 \cdot h = 0,3 \cdot (1,16) = 0,35\ \text{m} \le (0,10\ \text{m} + \textit{surfacing depth in metres}) = 0,20\ \text{m}$.

For $h \ge 0,8\ \text{m}$ we have $\Delta T_1 = 13,0°\text{C}$; $\Delta T_2 = 3,0°\text{C}$; $\Delta T_3 = 2,5°\text{C}$.

(b) Cooling

$h_1 = h_4 = 0,20 \cdot h = 0,20 \cdot (1,16) = 0,23\ \text{m} \le 0,25\ \text{m} \quad \rightarrow \quad h_1 = h_4 = 0,23\ \text{m}$

$h_2 = h_3 = 0,20 \cdot h = 0,25 \cdot (1,16) = 0,29\ \text{m} \le 0,20\ \text{m} \quad \rightarrow \quad h_2 = h_3 = 0,29\ \text{m}$.

Linear interpolation for ΔT_j within the range $1,0\ \text{m} < h < 1,5\ \text{m}$ with $h = 1,16\ \text{m}$:

h [m]	ΔT_1 [°C]	ΔT_2 [°C]	ΔT_3 [°C]	ΔT_4 [°C]
1,0	– 8,0	– 1,5	– 1,5	– 6,3
1,16	ΔT_1	ΔT_2	ΔT_3	ΔT_4
1,5	– 8,4	– 0,5	– 1,0	– 6,5

Table 34.11 Values from Figure 6.2c - Temperature differences for bridge decks - Type 3: Concrete decks.

$$\frac{(-8,4)-(-8,0)}{1,5-1,0} = \frac{\Delta T_1-(-8,0)}{1,16-1,0} \quad \rightarrow \quad \Delta T_1 = -8,13°\text{C}$$

$$\frac{(-0,5)-(-1,5)}{1,5-1,0} = \frac{\Delta T_2-(-1,5)}{1,16-1,0} \quad \rightarrow \quad \Delta T_2 = -1,18°\text{C}$$

$$\frac{(-1,0)-(-1,5)}{1,5-1,0} = \frac{\Delta T_3-(-1,5)}{1,16-1,0} \quad \rightarrow \quad \Delta T_3 = -1,34°\text{C}$$

$$\frac{(-6,5)-(-6,3)}{1,5-1,0} = \frac{\Delta T_4-(-6,3)}{1,16-1,0} \quad \rightarrow \quad \Delta T_4 = -6,36°\text{C}.$$

Rounded to the first decimal place we get:

$\Delta T_1 = -8,1°\text{C}$; $\Delta T_2 = -1,2°\text{C}$; $\Delta T_3 = -1,3°\text{C}$; $\Delta T_4 = -6,4°\text{C}$.

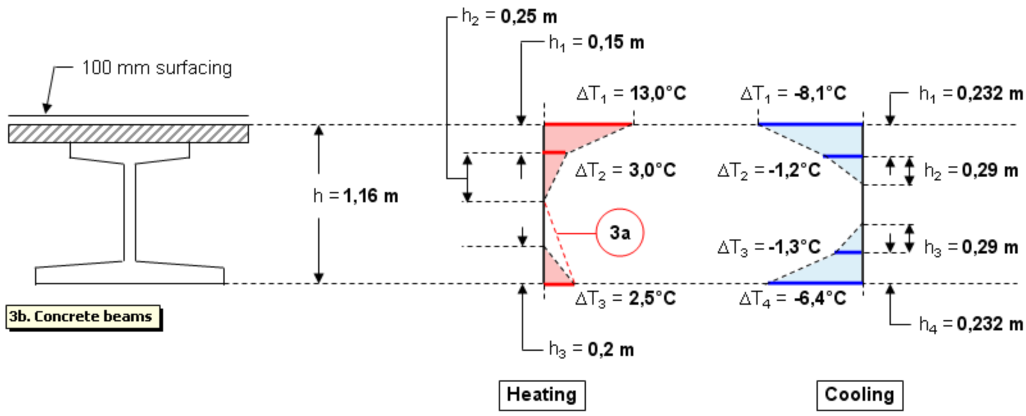


Figure 34.24 Excel® output graph (for Bridge deck Type 3c): characteristic values.

EXAMPLE 34-U- Characteristic thermal actions in bridges - Simultaneity of uniform and temperature difference components - **test4**

Given: Taking into account both the temperature difference $\Delta T_{M,\,heat}$ (or $\Delta T_{M,\,cool}$) and the maximum range of uniform bridge temperature component $\Delta T_{N,\,exp}$ (or $\Delta T_{N,\,con}$) assuming simultaneity, find the most adverse effect to be chosen as input in the FEM analysis. Refer to the data in Example 34-S (bridge deck Type 1 with $T_{e,\,min} = -21°C$, $T_{e,\,max} = 50°C$).

[*Reference sheet:* CodeSec6]-[*Cell-Range: A415:O415-A509:O509*].

Solution: From Expressions (6.1) and (6.2) we get the characteristic value of the maximum contraction and maximum expansion value of the uniform bridge temperature component respectively (bridge deck Type 1):

$$\Delta T_{N,\,con} = T_0 - T_{e,\,min} = [10-(-21)] = 31°C$$

$$\Delta T_{N,\,exp} = T_{e,\,max} - T_0 = [50-10] = 40°C\,.$$

From data in Example 34-S we have:

$$\Delta T_{M,\,heat} = 12,6°C \text{ (expansion);}$$

$$\Delta T_{M,\,cool} = 15,6°C \text{ (contraction).}$$

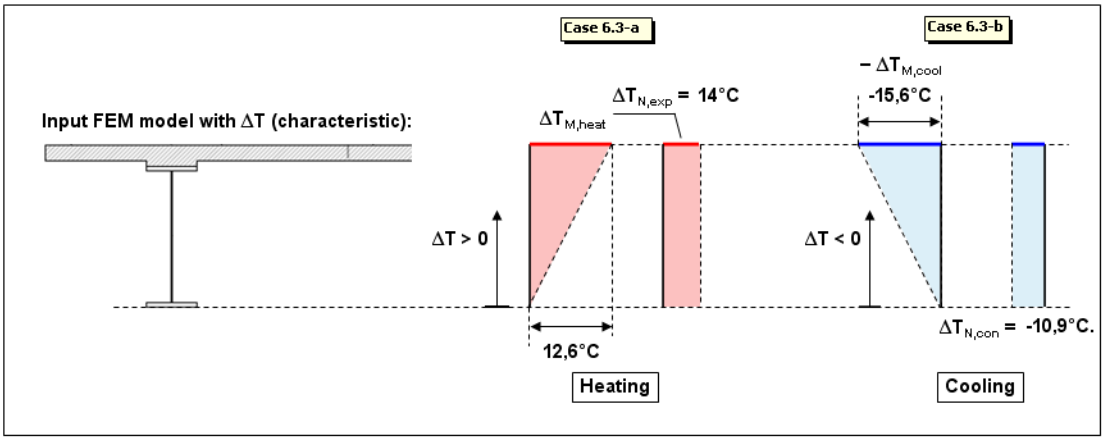
From Expressions (6.3) and (6.4), using the given numerical data, we get respectively:

$$\begin{array}{l} \text{Load} - \textit{Case 6.3-a:} \\ \text{Load} - \textit{Case 6.3-b:} \end{array} \begin{cases} \Delta T_{M,\,heat} + \omega_N \cdot \Delta T_{N,\,exp} = 12,6 + 0,35 \cdot (40) = (12,6+14)°C \\ \Delta T_{M,\,cool} + \omega_N \cdot \Delta T_{N,\,con} = -15,6 + 0,35 \cdot (-31) = -(15,6+10,9)°C \end{cases}$$

$$\begin{array}{l} \text{Load} - \textit{Case 6.4-a:} \\ \text{Load} - \textit{Case 6.4-b:} \end{array} \begin{cases} \omega_M \cdot \Delta T_{M,\,heat} + \Delta T_{N,\,exp} = 0,75 \cdot (12,6) + 40 = (9,45+40)°C \\ \omega_M \cdot \Delta T_{M,\,cool} + \Delta T_{N,\,con} = 0,75 \cdot (-15,6) + (-31) = (-11,7-31)°C \end{cases}$$

having assumed $\omega_N = 0,35$, $\omega_M = 0,75$ for the reduction factors.

Having thus considered four different combinations of load (Case 6.3-a; Case 6.3-b; Case 6.4-a; Case 6.4-b), we have (see Figure above):



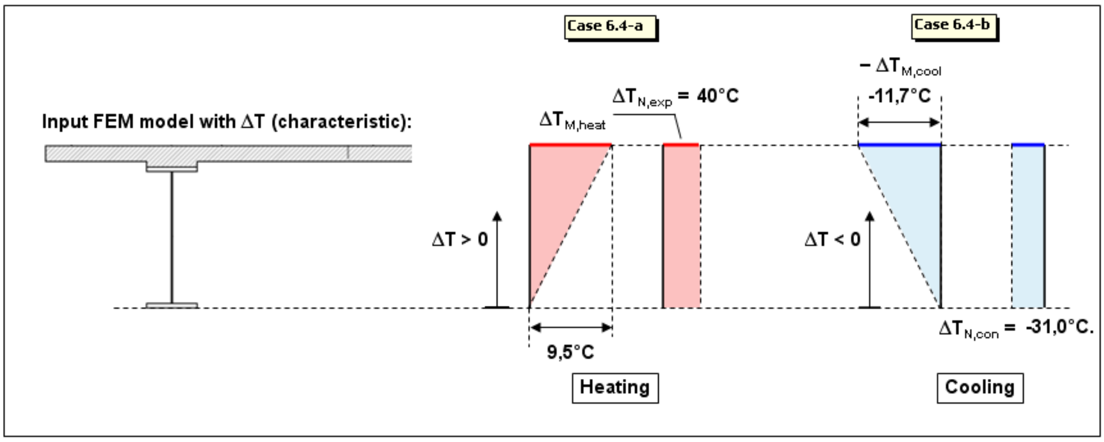


Figure 34.25 Excel® output graph (for Bridge deck Type 1): characteristic values.

34.4 References [Section 34]

EN 1991-1-5:2003. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, November 2003 (DAV)

EN 1991-1-5:2003/AC:2009. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2009

Manual for the design of building structures to Eurocode 1 and Basis of Structural Design April 2010. © 2010 The Institution of Structural Engineers

JRC Scientific and Technical Reports. Bridge Design to Eurocodes Worked examples. Worked examples presented at the Workshop "Bridge Design to Eurocodes", Vienna, 4-6 October 2010. Support to the implementation, harmonization and further development of the Eurocodes. Y. Bouassida, E. Bouchon, P. Crespo, P. Croce, L. Davaine, S. Denton, M. Feldmann, R. Frank, G. Hanswille, W. Hensen, B. Kolias, N. Malakatas, G. Mancini, M. Ortega, J. Raoul, G. Sedlacek, G. Tsionis.

Section 35 Eurocode 1 EN 1991-1-5 Annex A, Annex B

35.1 Annex A (Normative): Isotherms of national minimum and maximum shade air temperatures

35.1.1 General

The values of both annual minimum and annual maximum shade air temperature represent values with an annual probability of being exceeded of 0,02. Thermal actions must be considered to be variable and indirect actions. Regulations furnish characteristic values whose probability of being exceeded is 0,02, which is equivalent to a return period of 50 years. The fundamental quantities on which thermal actions are based are the extreme air temperatures, that is, the maximum and minimum, in the shade at the building site. Such values are furnished by the National Meteorological Institute of each Member State. Eurocode EN 1991-1-5 does not include maps of extreme temperatures. Such task is left up to the National Meteorological Institutes. Indicative maps for some CEN countries were included in the preliminary standard ENV 1991-2-5.

The initial temperature T_0 should be taken as the temperature of a structural element at the relevant stage of its restraint (completion). If it is not predictable the average temperature during the construction period should be taken. The value of T_0 may be specified in the National annex or in a particular project. If no information is available T_0 may be taken as 10°C. In case of uncertainty concerning sensitivity of the bridge to T_0, it is recommended that a lower and upper bound of an interval expected for T_0 are considered.

35.1.2 Maximum and minimum shade air temperature values with an annual probability of being exceeded p other than 0,02

If the value of maximum (or minimum) shade air temperature, $T_{max,p}$ ($T_{min,p}$), is based on an annual probability of being exceeded p other than 0,02 the ratios may be determinated from the following expressions based on a Generalized Extreme Value (GEV) Distribution (Type I: Gumbel):

— for maximum (T_{max}):

$$\frac{T_{max,p}}{T_{max}} = k_1 - k_2 \cdot \ln[-\ln(1-p)] \qquad \textbf{(Eq. 35-36)}$$

— for minimum (T_{min}):

$$\frac{T_{min,p}}{T_{min}} = k_3 + k_4 \cdot \ln[-\ln(1-p)], \qquad \textbf{(Eq. 35-37)}$$

where T_{min} (T_{max}) is the value of minimum (maximum) shade air temperature (at height above sea level $h \geq 0$) with an annual probability of being exceeded of 0,02. The National annex may specify the values of the coefficients k_1, k_2, k_3 and k_4. If no other information is available the following values are recommended:

$$k_1 = 0,781 ; \; k_2 = 0,056 ; \; k_3 = 0,393 ; \; k_4 = -0,156 . \qquad \textbf{(Eq. 35-38)}$$

Expression 35-37 can only be used if T_{min} is negative.

If specific data are available (mean "m" and the standard deviation "σ"of the type I extreme value distribution) then the following expressions shall be used:

— for maximum (T_{max}):

$$k_1 = \frac{uc}{uc + 3,902} \qquad \textbf{(Eq. 35-39)}$$

$$k_2 = \frac{k_1}{uc}, \qquad \textbf{(Eq. 35-40)}$$

with:

$$\begin{cases} u = m - 0,57722/c \\ c = 1,2825/\sigma \end{cases} \qquad \textbf{(Eq. 35-41)}$$

— for minimum (T_{min}):

$$k_3 = \frac{uc}{uc - 3,902} \qquad \textbf{(Eq. 35-42)}$$

$$k_4 = \frac{k_3}{uc} \qquad \textbf{(Eq. 35-43)}$$

with:

$$\begin{cases} u = m + 0,57722/c \\ c = 1,2825/\sigma \end{cases} \qquad \textbf{(Eq. 35-44)}$$

35.2 Annex B (Normative): Temperature differences for various surfacing depths

Temperature difference profiles given in Figures 6.2a 6.2c are valid for 40 mm surfacing depths for deck type 1 and 100 mm surfacing depths for types 2 and 3. The National annex may give values for other depths. Recommended values are given in the following tables: Table B.1 (for Type 1), B.2 (for Type 2) and B.3 (for Type 3).

Surfacing thickness	Temperature difference [°C]				
	Heating				Cooling
	ΔT_1	ΔT_2	ΔT_3	ΔT_4	ΔT_1
unsurfaced	30	16	6	3	8
20 mm	27	15	9	5	6
40 mm	24	14	8	4	6

Table 35.12 From Table B.1 - Recommended values of ΔT for deck Type 1.

Depth of slab	Surface thickness	Temperature difference [°C]	
		Heating	Cooling
		ΔT_1	ΔT_1
h = 0,2 m	unsurfaced	16,5	5,9
	waterproofed[(a)]	23,0	5,9
	50 mm	18,0	4,4
	100 mm	13,0	3,5
	150 mm	10,5	2,3
	200 mm	8,5	1,6
		ΔT_1	ΔT_1
h = 0,3 m	unsurfaced[(a)]	18,5	9,0
	waterproofed	26,5	9,0
	50 mm	20,5	6,8
	100 mm	16,0	5,0
	150 mm	12,5	3,7
	200 mm	10,0	2,7

Table 35.13 From Table B.2 - Recommended values of ΔT for deck Type 2.

(a). These values represent upper bound values for dark colour.

Depth of slab [m]	Surfacing thickness	Temperature difference [°C]						
		Heating			Cooling			
		ΔT_1	ΔT_2	ΔT_3	ΔT_1	ΔT_2	ΔT_3	ΔT_4
0,2	unsurfaced	12,0	5,0	0,1	4,7	1,7	0,0	0,7
	waterproofed[(a)]	19,5	8,5	0,0	4,7	1,7	0,0	0,7
	50 mm	13,2	4,9	0,3	3,1	1,0	0,2	1,2
	100 mm	8,5	3,5	0,5	2,0	0,5	0,5	1,5
	150 mm	5,6	2,5	0,2	1,1	0,3	0,7	1,7
	200 mm	3,7	2,0	0,5	0,5	0,2	1,0	1,8
0,4	unsurfaced	15,2	4,4	1,2	9,0	3,5	0,4	2,9
	waterproofed[(a)]	23,6	6,5	1,0	9,0	3,5	0,4	2,9
	50 mm	17,2	4,6	1,4	6,4	2,3	0,6	3,2
	100 mm	12,0	3,0	1,5	4,5	1,4	1,0	3,5
	150 mm	8,5	2,0	1,2	3,2	0,9	1,4	3,8
	200 mm	6,2	1,3	1,0	2,2	0,5	1,9	4,0
0,6	unsurfaced	15,2	4,0	1,4	11,8	4,0	0,9	4,6
	waterproofed[(a)]	23,6	6,0	1,4	11,8	4,0	0,9	4,6
	50 mm	17,6	4,0	1,8	8,7	2,7	1,2	4,9
	100 mm	13,0	3,0	2,0	6,5	1,8	1,5	5,0
	150 mm	9,7	2,2	1,7	4,9	1,1	1,7	5,1
	200 mm	7,2	1,5	1,5	3,6	0,6	1,9	5,1
0,8	unsurfaced	15,4	4,0	2,0	12,8	3,3	0,9	5,6
	waterproofed[(a)]	23,6	5,0	1,4	12,8	3,3	0,9	5,6
(Cont'd)	50 mm	17,8	4,0	2,1	9,8	2,4	1,2	5,8
	100 mm	13,5	3,0	2,5	7,6	1,7	1,5	6,0
	150 mm	10,0	2,5	2,0	5,8	1,3	1,7	6,2
	200 mm	7,5	2,1	1,5	4,5	1,0	1,9	6,0

Table 35.14 From Table B.3 - Recommended values of ΔT for deck Type 3.

Depth of slab [m]	Surfacing thickness	Temperature difference [°C]						
(Cont'd)	unsurfaced	15,4	4,0	2,0	13,4	3,0	0,9	6,4
	waterproofed[(a)]	23,6	5,0	1,4	13,4	3,0	0,9	6,4
	50 mm	17,8	4,0	2,1	10,3	2,1	1,2	6,3
1,0	100 mm	13,5	3,0	2,5	8,0	1,5	1,5	6,3
	150 mm	10,0	2,5	2,0	6,2	1,1	1,7	6,2
	200 mm	7,5	2,1	1,5	4,3	0,9	1,9	5,8
	unsurfaced	15,4	4,5	2,0	13,7	1,0	0,6	6,7
	waterproofed[(a)]	23,6	5,0	1,4	13,7	1,0	0,6	6,7
	50 mm	17,8	4,0	2,1	10,6	0,7	0,8	6,6
1,5	100 mm	13,5	3,0	2,5	8,4	0,5	1,0	6,5
	150 mm	10,0	2,5	2,0	6,5	0,4	1,1	6,2
	200 mm	7,5	2,1	1,5	5,0	0,3	1,2	5,6
		ΔT_1	ΔT_2	ΔT_3	ΔT_1	ΔT_2	ΔT_3	ΔT_4
		Heating			**Cooling**			

Table 35.14 From Table B.3 - Recommended values of ΔT for deck Type 3.

(a). These values represent upper bound values for dark colour.

35.3 Verification tests

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— Splash
— AnnexA
— AnnexB.

EXAMPLE 35-V- Isotherms of national minimum and maximum shade air temperatures - **test1**

Given: The values of maximum and minimum shade air temperature at sea level (h = 0) with an annual probability of being exceeded of 0,02 are given respectively:

– T_{max} = 34°C (rounded value for probability p = 0,02)

– T_{min} = – 18°C (rounded value for probability p = 0,02).

Find the values of maximum and minimum shade air temperatures for an annual probability of being exceeded p equivalent to a return period of 50 years for an height above sea level $\Delta h = 150$ m.

[*Reference sheet:* AnnexA]-[*Cell-Range: A1:O1-A23:O23*].

Solution: If no information is available the values of shade air temperature may be adjusted for height above sea level by subtracting 0,5°C per 100 m height for minimum shade air temperatures and 1,0°C per 100 m height for maximum shade air temperatures. Therefore, for a return period of $T = 50$ years $p = 1/T = 1/50 = 0,02$ we get:

– for minimum:

$$\Delta T_{min} = \Delta h \times \left(\frac{-0,5°C}{100\ m}\right) = (150\ m) \times \left(\frac{-0,5°C}{100\ m}\right) = -0,75°C$$

$$T_{min,\,p} = T_{min} + \Delta T_{min} = (-18) + (-0,75) = -18,75°C\,.$$

– for maximum:

$$\Delta T_{max} = \Delta h \times \left(\frac{-1,0°C}{100\ m}\right) = (150\ m) \times \left(\frac{-1,0°C}{100\ m}\right) = -1,50°C$$

$$T_{max,\,p} = T_{max} + \Delta T_{max} = (34) + (-1,50) = 32,50°C\,.$$

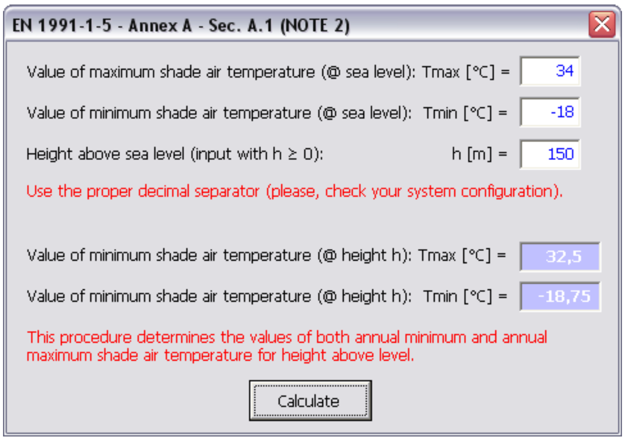

Figure 35.26 PreCalculus Excel® form: procedure for a quick pre-calculation.

EXAMPLE 35-W- Isotherms of national minimum and maximum shade air temperatures - **test1b**

Given: Assuming the same assumptions from the previous example find the values of maximum $T_{max,\,p}$ and minimum $T_{min,\,p}$ shade air temperature based on an annual probability of being exceeded p equivalent to a return period of 90 years for an height above sea level Δh = 150 m. Let us assume that the mean value "m" and the standard deviation "σ" of a Generalized Extreme Value (GEV) Distribution (Type I: Gumbel) are respectively:

– for maximum temperatures:

$$m(T_{max}) = 34°C\,;\ \sigma(T_{max}) = 3°C\,.$$

– for minimum temperatures:

$m(T_{min}) = -7°C$; $\sigma(T_{min}) = 3°C$.

Find the ratios $T_{max,p}/T_{max}$, $T_{min,p}/T_{min}$ against the annual probability p of being exceeded within the range [0,005; 0,5].

[*Reference sheet:* AnnexA]-[*Cell-Range: A26:O26-A167:O167*].

Solution: From Expressions (A.7) and (A.8):

– for maximum temperatures:

$$\begin{cases} u = m - 0,57722/c = 34 - 0,57722/0,43 = 32,66°C \\ c = 1,2825/\sigma = 1,2825/3 = 0,43 \end{cases}$$

– for minimum temperatures:

$$\begin{cases} u = m + 0,57722/c = (-7) + 0,57722/0,43 = -5,65°C \\ c = 1,2825/\sigma = 1,2825/3 = 0,43 \end{cases}$$

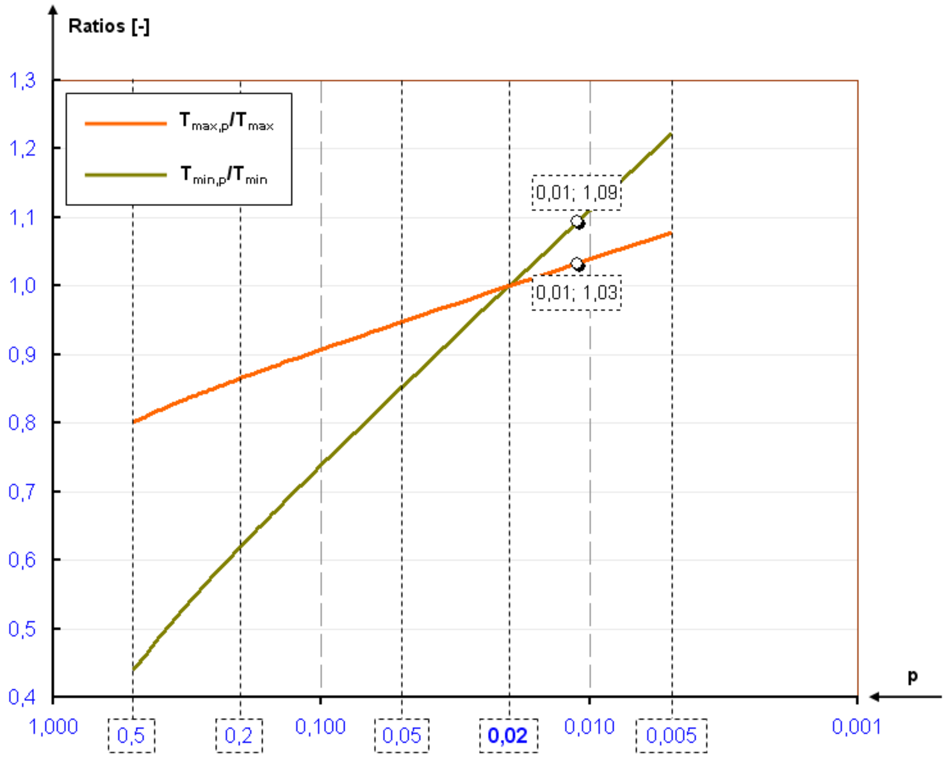

Figure 35.27 Excel® output graph (from Figure A.1).

It moreover follows that:

– for maximum temperatures:

$$uc = (32,66) \cdot (0,43) = 14,04 \, [-]$$

$$k_1 = \frac{uc}{uc + 3,902} = \frac{14,04}{14,04 + 3,902} = 0,78\ [\text{-}];\ k_2 = \frac{k_1}{uc} = \frac{0,78}{14,04} = 0,06\ [\text{-}]$$

– for minimum temperatures:

$$uc = (-5,65) \cdot (0,43) = -2,43\ [\text{-}].$$

$$k_3 = \frac{uc}{uc - 3,902} = \frac{(-2,43)}{(-2,43) - 3,902} = 0,38\ [\text{-}];\ k_4 = \frac{k_3}{uc} = \frac{0,38}{(-2,43)} = -0,16\ [\text{-}].$$

Values of maximum and minimum shade air temperature

(annual probability of being exceeded p = 0,011)

– for maximum (rounded value):

$$\frac{T_{max,p}}{T_{max}} = k_1 - k_2 \cdot \ln[-\ln(1-p)] = 0,78 - 0,06 \cdot \ln[-\ln(1-0,011)] = 1,0\ [\text{-}]$$

– for minimum: (rounded value):

$$\frac{T_{min,p}}{T_{min}} = k_3 + k_4 \cdot \ln[-\ln(1-p)] = 0,38 + (-0,16) \cdot \ln[-\ln(1-0,011)] = 1,1\ [\text{-}]$$

For the ratios $T_{max,p}/T_{max}$, $T_{min,p}/T_{min}$ against the annual probability p of being exceeded within the range [0,005; 0,5] see Figure 35.27 above.

example-end

35.4 References [Section 35]

EN 1991-1-5:2003. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, November 2003 (DAV)

EN 1991-1-5:2003/AC:2009. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2009

Implementation of Eurocodes - Handbook 3 - Action effects for buildings. Guide to basis of structural reliability and risk engineering related to Eurocodes supplemented by practical examples. LEONARDO DA VINCI PILOT PROJECT CZ/02/B/F/PP-134007. Aachen 10.2005

Thermal Actions. Czech Technical University in Prague, Czech Republic. Milan Holický and Jana Marková. 2013.

Section 36 Eurocode 1 EN 1991-1-5 Annex D

36.1 Annex D (Informative): Temperature profiles in buildings and other construction works

36.1.1 General

Temperature profiles may be determined using the thermal transmission theory. In the case of a simple sandwich element (e.g. slab, wall, shell) under the assumption that local thermal bridges do not exist a temperature T(x) at a distance x from the inner surface of the cross section may be determined assuming steady thermal state as:

$$T(x) = T_{in} - \frac{dQ}{dt} \cdot R(x) = T_{in} - \frac{(T_{in} - T_{out})}{R_{tot}} \cdot R(x) \qquad \textbf{(Eq. 36-45)}$$

where:

- $T_{in}[°C]$ is the air temperature of the inner environment
- $T_{out}[°C]$ is the temperature of the outer environment
- $R_{tot}[°C/W]$ is the total thermal resistance (of part) of the element (e.g. wall or window) including resistance of both surfaces
- $R(x)[°C/W]$ is the thermal resistance at the inner surface and of (part of) the element from the inner surface up to the point “x”.

The resistance values R_{tot} and $R(x)$ may be determined using the coefficient of heat transfer and coefficients of thermal conductivity given in EN ISO 6946 (1996) and EN ISO 13370 (1998):

$$R_{tot} = R_{in} + \sum_i \frac{h_i}{\lambda_i} + R_{out} = R_{in} + \left(\frac{h_1}{\lambda_1 \cdot A_1} + \frac{h_2}{\lambda_2 \cdot A_2} + \ldots + \frac{h_N}{\lambda_N \cdot A_N}\right) + R_{out} \qquad \textbf{(Eq. 36-46)}$$

where:

- $R_{in}[°C/W]$ is the thermal resistance at the inner surface (of part) of the element
- $R_{out}[°C/W]$ is the thermal resistance at the outer surface (of part) of the element
- N is the number of layers between the inner and the outer surfaces
- $\lambda_i[W/(m \cdot °C)]$ is the thermal conductivity of the layer-i
- $h_i[m]$ is the thickness of the layer-i
- $A_i[m^2]$ is the heat transfer surface considered in the calculations (and part) of the entire actual surface $A \geq A_i$ (e.g. wall or window).

Hence:

$$R(x) = R_{in} + \left(\frac{h_1}{\lambda_1 \cdot A_1} + \frac{h_2}{\lambda_2 \cdot A_2} + \ldots + \frac{h_i}{\lambda_i \cdot A_i}\right) \quad \text{(Eq. 36-47)}$$

where layers (or part of a layer) from the inner surface up to point "x" are considered only.

Note Thermal resistance in buildings: 0,10 to 0,17 [m^2 °C/W] (depending on the orientation of the heat flow), and R_{out} = 0,04 [m^2 °C/W] (for all orientations). The thermal conductivity λ_i for concrete (of volume of weight from 21 to 25 kN/m^3) varies from 1,16 to 1,71 W/(m°C).

36.2 Verification tests

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— Splash

— AnnexC

— AnnexD.

EXAMPLE 36-X- Temperature distribution within a wall of a building - **test1**

Given: Find the thermal resistance and the temperature distribution within a wall (see Figure below) assuming one-dimensional steady-state heat transfer. Determine the thermal power (heat flow rate dQ_{tot}/dt) transmitted through the entire wall.

Let us assume the following assumptions:

– heat transfer coefficient at the inner surface: h_i = 10 W/m^2°C

– air temperature of the inner environment: $T_{in} = 20°C$

– heat transfer coefficient at the outer surface: h_{out} = 25 W/m^2°C

– air temperature of the outer environment: $T_{out} = -15°C$

– thermal conductivity of the layer 1 (*gypsum*): $\lambda_1 = 0,17\ W/(m^2 \cdot °C)$
– thermal conductivity of the layer 2 (*glass fibre batt*): $\lambda_2 = 0,045\ W/(m^2 \cdot °C)$
– thermal conductivity of the layer 3 (*plywood*): $\lambda_3 = 0,13\ W/(m^2 \cdot °C)$
– thermal conductivity of the layer 4 (*metal siding*): $\lambda_4 = 0,10\ W/(m^2 \cdot °C)$
– wall 3 meters high and 5 meters wide (actual surface $A = 15,00\ m^2$).
See in the picture below for geometric details.
[*Reference sheet:* AnnexA]-[*Cell-Range: A1:O1-A191:O191*].

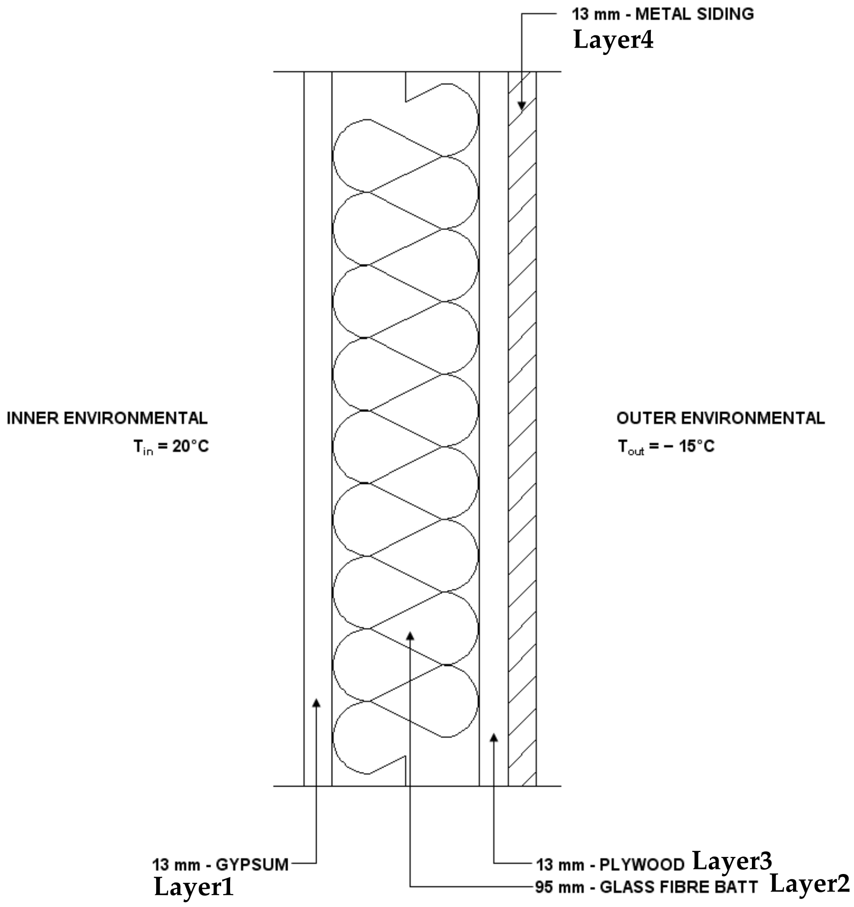

Figure 36.28A wall assembly.

Solution: Taking $A_j = 1,00\ m^2$ as heat transfer area to be used in calculations and using the given numerical data we find:

$$R_{in} = \frac{1}{h_{in} \cdot A_j} = \frac{1}{(10) \cdot 1} = 0,10\ °C/W$$

$$R_{out} = \frac{1}{h_{out} \cdot A_j} = \frac{1}{(25) \cdot 1} = 0,04\ °C/W$$

$$R_1 = \frac{h_1}{\lambda_1 \cdot A_1} = \frac{(13/10^3)}{(0,17) \cdot (1,00)} = 0,076\ °C/W$$

$$R_2 = \frac{h_2}{\lambda_2 \cdot A_2} = \frac{(95/10^3)}{(0,045) \cdot (1,00)} = 2,11\ °C/W$$

$$R_3 = \frac{h_3}{\lambda_3 \cdot A_3} = \frac{(13/10^3)}{(0,13) \cdot (1,00)} = 0,10\ °C/W$$

$$R_4 = \frac{h_4}{\lambda_4 \cdot A_4} = \frac{(13/10^3)}{(0,10) \cdot (1,00)} = 0,13\ °C/W\,.$$

Resistance value (see Eq. D.2):

$$R_{tot} = R_{in} + \left(\frac{h_1}{\lambda_1 \cdot A_1} + \frac{h_2}{\lambda_2 \cdot A_2} + \ldots + \frac{h_N}{\lambda_N \cdot A_N}\right) + R_{out} = 0,10 + (0,076 + 2,11 + 0,10 + 0,13 + 0,04)$$

$$R_{tot} = 2,56\ °C/W\ \text{(rounded value)}.$$

Heat flow rate

Heat flow rate dQ_{tot}/dt transmitted through the entire wall:

$$\frac{dQ_{tot}}{dt} = \left\{\left[\frac{(T_{in} - T_{out})}{R_{tot}}\right]/A_j\right\} \cdot A = \left\{\left[\frac{(20°C) - (-15°C)}{(2,56\ °C/W)}\right]/(1,00\ m^2)\right\} \cdot (15,00\ m^2)$$

$$\frac{dQ_{tot}}{dt} = \left(\frac{dQ}{dt}\frac{1}{A_j}\right) \cdot A = \left(13,67\ \frac{W}{m^2}\right) \cdot (15,00\ m^2) = 205\ W\,.$$

Temperatures distribution within the wall

Layer 1 (*gypsum*):

$$T_1 = T_{in} - \left(\frac{dQ}{dt} \cdot R_{in}\right) = 20 - 13,67 \cdot (0,10) = 18,6°C$$

Layer 2 (*glass fibre batt*):

$$T_2 = T_{in} - \frac{dQ}{dt} \cdot (R_{in} + R_1) = 20 - 13,67 \cdot (0,10 + 0,076) = 17,6°C$$

Layer 3 (*plywood*):

$$T_3 = T_{in} - \frac{dQ}{dt} \cdot (R_{in} + R_1 + R_2) = 20 - 13,67 \cdot (0,10 + 0,076 + 2,11) = -11,2°C$$

Layer 4 (*metal siding*):

$$T_4 = T_{out} + \frac{dQ}{dt} \cdot (R_{out} + R_4) = -15 + 13,67 \cdot (0,04 + 0,13) = -12,7°C$$

Outer surface:

$$T_5 = T_{out} + \left(\frac{dQ}{dt} \cdot R_{out}\right) = -15 + 13,67 \cdot (0,04) = -14,5°C\,.$$

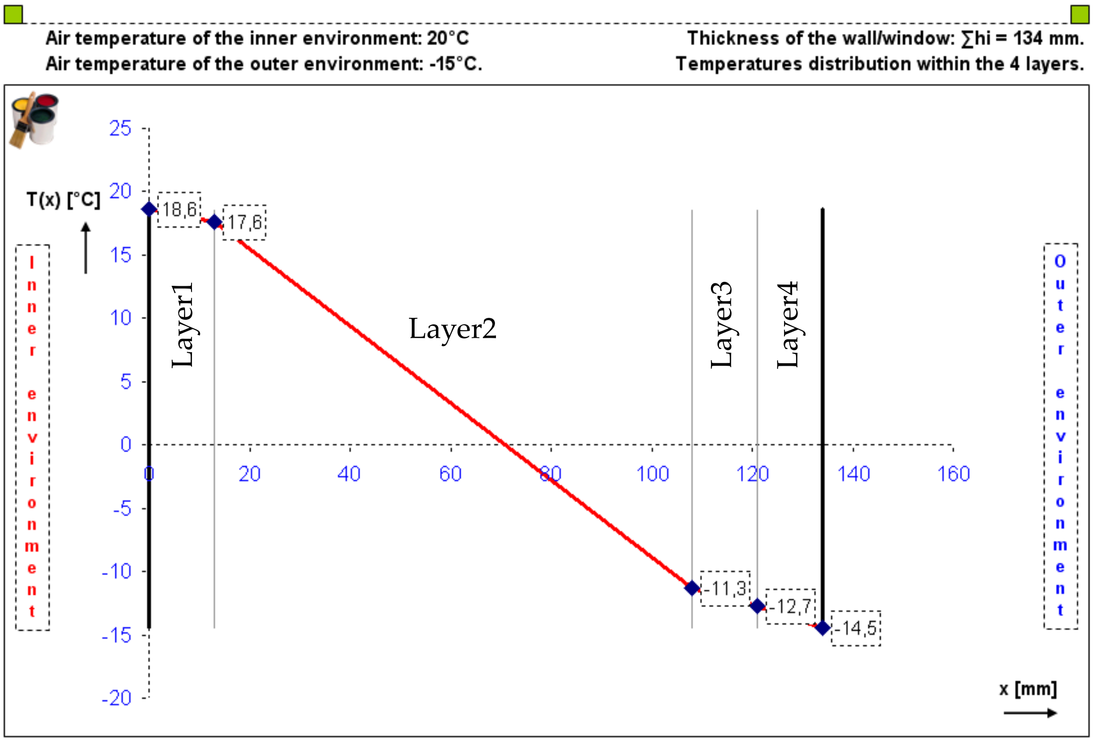

Figure 36.29 Excel® output graph (according to example Figure D.1 in Annex D).

Final step. Mark the temperatures at each component edge (interface), and then draw straight lines joining each point to the next.

This completes both the arithmetic and graphic representations of the temperature or thermal gradient (see Figure above).

example-end

EXAMPLE 36-Y- Temperature distribution within a wall of a building - **test1b**

Given: A wall 3 meters high and 5 meters wide is made up with long horizontal bricks of size 16 cm x 22 cm (cross section), separated by horizontal layers of mortar (thickness 3 cm). The bricks are covered by two vertical layers of mortar of thickness 2 cm each and finally by an outer insulating material (thickness 3 cm).

Find the thermal resistance and the temperature distribution within the wall assuming one-dimensional steady-state heat transfer. Let us assume the following assumptions:

– heat transfer coefficient at the inner surface: $h_i = 10$ W/m^2°C

– air temperature of the inner environment: $T_{in} = 20$°C

– heat transfer coefficient at the outer surface: $h_{out} = 25$ W/m^2°C

– air temperature of the outer environment: $T_{out} = -10°C$
– thermal conductivity of the layer 1 (*insulating material*): $\lambda_1 = 0,026\ W/(m^2 \cdot °C)$
– thermal conductivity of the layer 2 (*mortar*): $\lambda_2 = 0,22\ W/(m^2 \cdot °C)$
– thermal conductivity of the layer 3 (*brick*): $\lambda_3 = 0,72\ W/(m^2 \cdot °C)$
– thermal conductivity of the layer 4 (*mortar*): $\lambda_4 = 0,22\ W/(m^2 \cdot °C)$
– wall 3 meters high and 5 meters wide (actual surface $A = 15,00\ m^2$).

See in the picture below for geometric details.

[*Reference sheet:* AnnexA]-[*Cell-Range: A1:O1-A191:O191*]

Solution: Let us consider a wall surface portion A_j (see Figure below) with an height of $d_{j1} + d_{j2} + d_{j3}$ = 0,25 meters for 1 meter deep, since it is representative of the entire wall (thermally).

Hence, taking $A_j = (0,25\ m)(1,00\ m) = 0,25\ m^2$ as heat transfer area to be used in calculations and using the given numerical data we find:

$$R_{in} = \frac{1}{h_{in} \cdot A_j} = \frac{1}{(10) \cdot (0,25)} = 0,4\ °C/W$$

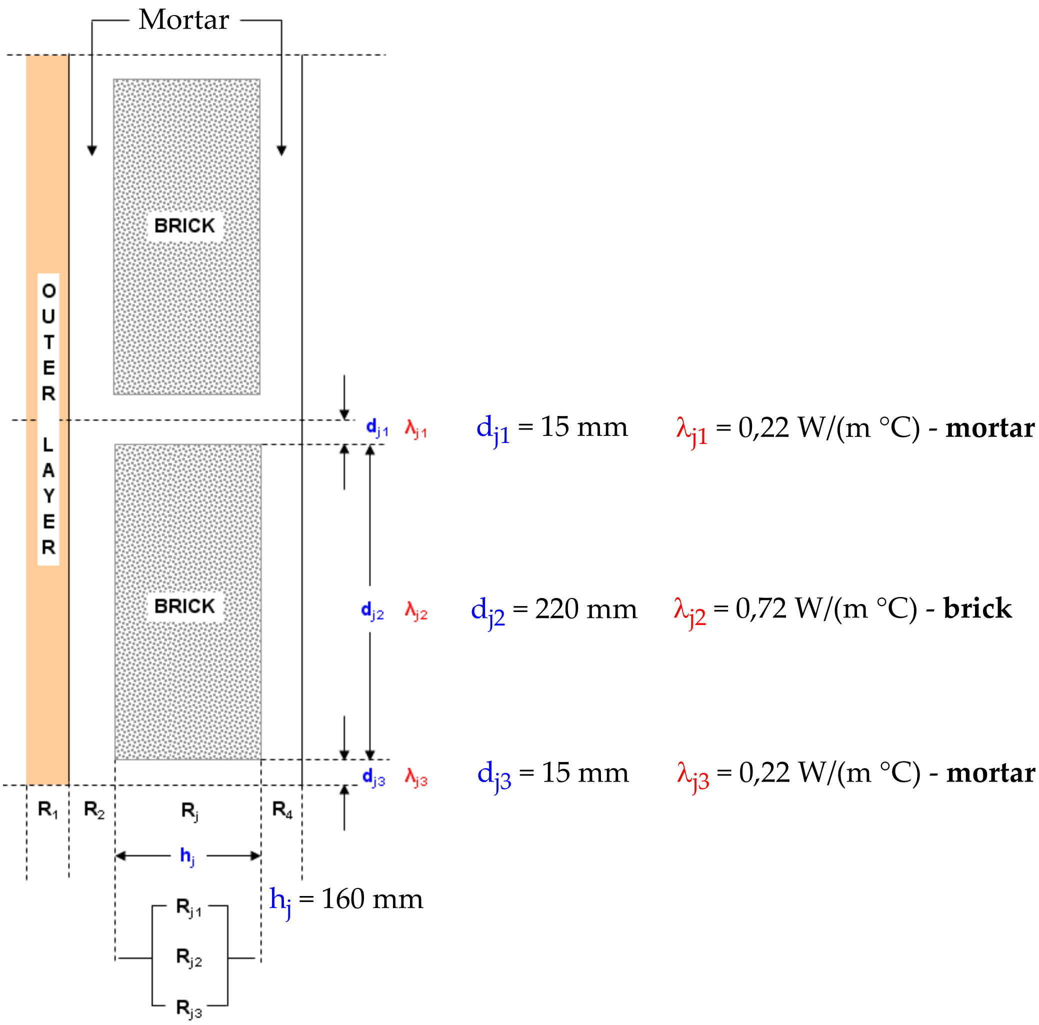

Figure 36.30 Wall with four vertical layers and interior and exterior films. Thermal network also shown.

$$R_{out} = \frac{1}{h_{out} \cdot A_j} = \frac{1}{(25) \cdot (0,25)} = 0,16\ °C/W$$

Layer 1 (*outer layer*):

$$R_1 = \frac{h_1}{\lambda_1 \cdot A_1} = \frac{(30/10^3)}{(0,026) \cdot (0,25)} = 4,61\ °C/W$$

Layer 2 and Layer 4 (*mortar*):

$$R_4 = R_2 = \frac{h_2}{\lambda_2 \cdot A_2} = \frac{(20/10^3)}{(0,22) \cdot (0,25)} = 0,36\ °C/W$$

Layer 3 (*mortar* + *brick*) - parallel thermal network model:

$$R_j = \frac{1}{\frac{1}{R_{j1}} + \frac{1}{R_{j2}} + \frac{1}{R_{j3}}}$$ with (see Figure 36.30):

$$R_{j1} = \frac{h_j}{\lambda_{j1} \cdot d_{j1}} = \frac{160}{0,22 \cdot 15} = 48,48\ °C/W$$

$$R_{j2} = \frac{h_j}{\lambda_{j2} \cdot d_{j2}} = \frac{160}{0,72 \cdot 220} = 1,01\ °C/W$$

$$R_{j3} = \frac{h_j}{\lambda_{j3} \cdot d_{j3}} = \frac{160}{0,22 \cdot 15} = 48,48\ °C/W\,.$$

Therefore, we get (for j = 3):

$$R_j = \frac{1}{\frac{1}{R_{j1}} + \frac{1}{R_{j2}} + \frac{1}{R_{j3}}} = \frac{1}{\frac{1}{48,48} + \frac{1}{1,01} + \frac{1}{48,48}} = 0,97\ °C/W\,.$$

Note For input in TABLE-2 (see sheet "AnnexD") we use the equivalent value $\lambda_{j,eq}$:

$$\lambda_{j,\,eq} = \frac{h_j}{R_j \cdot (d_{j1} + d_{j2} + d_{j3})} = \frac{160}{0,97 \cdot (15 + 220 + 15)} = 0,66\ W/(m \cdot °C)\,.$$

Resistance value (see Eq. D.2):

$$R_{tot} = R_{in} + (R_1 + R_2 + R_{j\,=\,3} + R_4) + R_{out} = 0,4 + (4,61 + 0,36 + 0,97 + 0,36) + 0,16$$

$$R_{tot} = 6,86\ °C/W \text{ (rounded value).}$$

Heat flow rate

Heat flow rate dQ_{tot}/dt transmitted through the entire wall:

$$\frac{dQ_{tot}}{dt} = \left\{\left[\frac{(T_{in} - T_{out})}{R_{tot}}\right]/A_j\right\} \cdot A = \left\{\left[\frac{(20°C) - (-10°C)}{(6,86\ °C/W)}\right]/(0,25\ m^2)\right\} \cdot (15,00\ m^2)$$

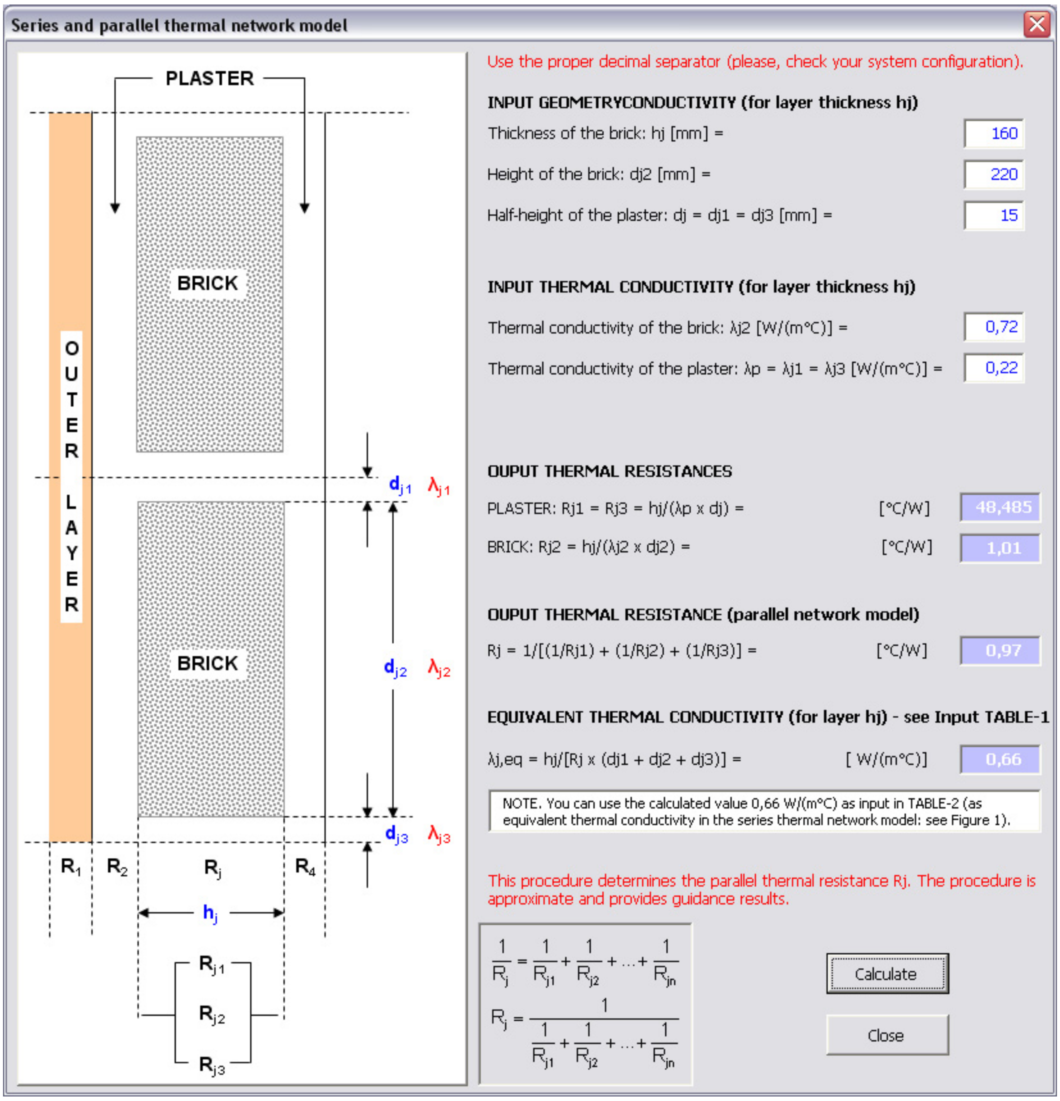

Figure 36.31 PreCalculus Excel® form: procedure for a quick pre-calculation.

$$\frac{dQ_{tot}}{dt} = \left(\frac{dQ}{dt}\frac{1}{A_j}\right) \cdot A = \left(17,50\ \frac{W}{m^2}\right) \cdot (15,00\ m^2) = 262\ W\,,$$

with: $\frac{dQ}{dt} = \frac{(T_{in} - T_{out})}{R_{tot}} = \frac{(20°C) - (-10°C)}{(6,86\ °C/W)} = 4,37\ W$ for the surface area A_j used for thermal calculations.

Temperatures distribution within the wall

Layer 1 (*outer layer*):

$$T_1 = T_{in} - \left(\frac{dQ}{dt} \cdot R_{in}\right) = 20 - 4,37 \cdot (0,40) = 18,3°C$$

Layer 2 (*mortar*):

$$T_2 = T_{in} - \frac{dQ}{dt} \cdot (R_{in} + R_1) = 20 - 4,37 \cdot (0,40 + 4,61) = -1,9°C$$

Layer 3 (*mortar + brick - parallel thermal network model*):

$$T_3 = T_{in} - \frac{dQ}{dt} \cdot (R_{in} + R_1 + R_2) = 20 - 4,37 \cdot (0,40 + 4,61 + 0,36) = -3,5°C$$

Layer 4 (*mortar*):

$$T_4 = T_{out} + \frac{dQ}{dt} \cdot (R_{out} + R_4) = -10 + 4,37 \cdot (0,16 + 0,36) = -7,7°C$$

Outer surface:

$$T_5 = T_{out} + \left(\frac{dQ}{dt} \cdot R_{out}\right) = -10 + 4,37 \cdot (0,16) = -9,3°C\,.$$

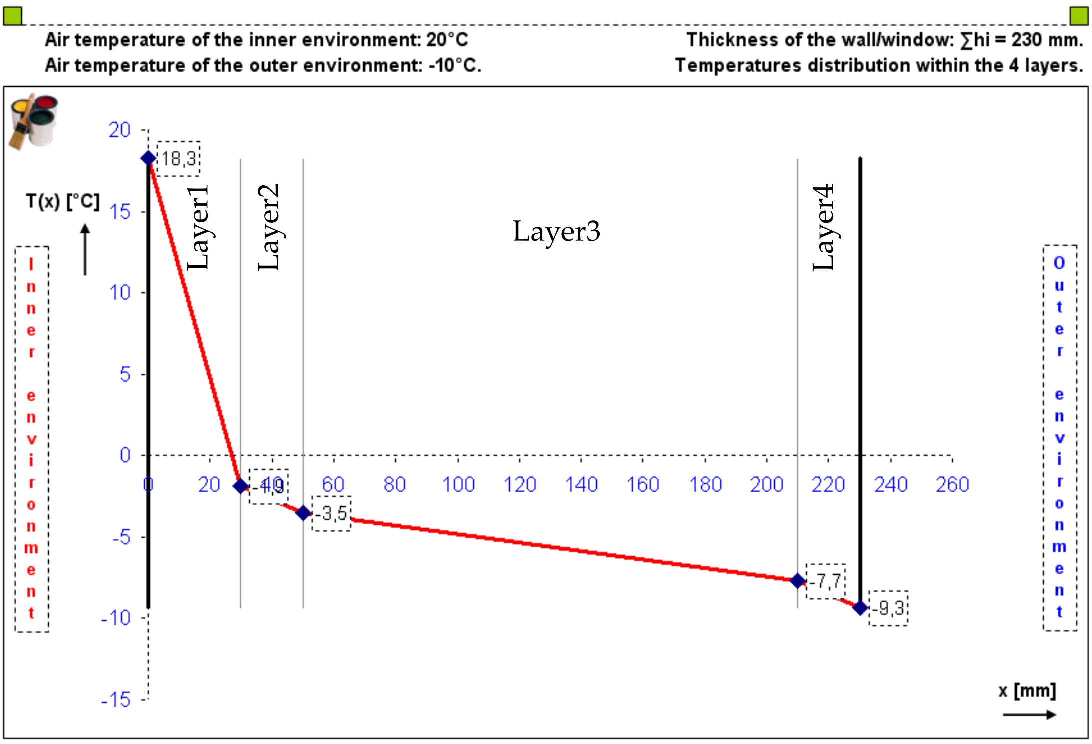

Figure 36.32 Excel® output graph (according to example Figure D.1 in Annex D).

This example demonstrates calculation of the thermal resistance and temperature distribution within a wall assuming one-dimensional steady-state heat transfer. Note that in some cases different parts of the wall may have different layers (in this case see layer No. 3).

To determine a correct wall R-value in such cases, we need to calculate the correct value through each heat flow path and determine the overall R-value based on the relative area of each path.

36.3 References [Section 36]

EN 1991-1-5:2003. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, November 2003 (DAV)

EN 1991-1-5:2003/AC:2009. Eurocode 1: Actions on structures - Part 1-5: General actions - Thermal actions. Brussels: CEN/TC 250 - Structural Eurocodes, March 2009

Implementation of Eurocodes - Handbook 3 - Action effects for buildings. Guide to basis of structural reliability and risk engineering related to Eurocodes supplemented by practical examples. LEONARDO DA VINCI PILOT PROJECT CZ/02/B/F/PP-134007. Aachen 10.2005

Thermal Actions. Czech Technical University in Prague, Czech Republic. Milan Holický and Jana Marková. 2013.

Section 1 Eurocode 1 EN 1991-1-6

1.1 General

This part of EN 1991 provides principles and general rules for the determination of actions which should be taken into account during the execution of buildings and civil engineering works.

In EN 1991-1-6, actions during execution are separated, according to their origin and in conformity with EN 1990, in Construction loads and Non construction loads. Actions during execution which include, where appropriate, construction loads and those other than construction loads shall be classified in accordance with EN 1990:2002, 4.1.1.

Construction loads (see also Sec. 4.11) should be classified as variable actions (Q_c). Table 4.1 gives the full description and classification of construction loads:

— Personnel and hand tools (variable)[1] (free)

— Storage movable items (variable) (free)

— Non-permanent equipment (variable) (fixed/free)[2]

— Movable heavy machinery and equipment (variable) (free)

— Accumulation of waste materials (variable) (free)

— Loads from parts of structure in temporary states (variable) (free).

Construction loads, which are caused by cranes, equipment, auxiliary construction works/structures may be classified as fixed or free actions depending on the possible position(s) for use.

The limits may be defined in the National Annex and for the individual project. In accordance with EN 1990:2002, 1.3(2), control measures may have to be adopted to verify the conformity of the position and moving of construction loads with the design assumptions.

(1) Variation in time: variable.

(2) Where construction loads are classified as fixed, then tolerances for possible deviations from the theoretical position should be defined. Where construction loads are classified as free, then the limits of the area where they may be moved or positioned should be determined.

1.2 Design situations and limit states

DESIGN SITUATIONS. Transient, accidental and seismic design situations shall be identified and taken into account as appropriate for designs for execution. Design situations should be selected as appropriate for the structure as a whole, the structural members, the partially completed structure, and also for auxiliary construction works and equipment. The selected design situations shall take into account the conditions that apply from stage to stage during execution in accordance with EN 1990:2002, 3.2(3)P.

The selected design situations shall be in accordance with the execution processes anticipated in the design. Design situations shall take account of any revisions to the execution processes. Any selected transient design situation should be associated with a nominal duration equal to or greater than the anticipated duration of the stage of execution under consideration. The design situations should take into account the likelihood for any corresponding return periods of variable actions (e.g. climatic actions).

Note The return periods for the determination of characteristic values of variable actions during execution may be defined in the National Annex or for the individual project. Recommended return periods for climatic actions are given in table 3.1, depending on the nominal duration of the relevant design situation.

Duration	Return period [years]
≤ 3 days	2[a]
≤ 3 months (but > 3 days)	5[b]
≤ 1 year (but > 3 months)	10
> 1 year	50

Table 1.15 From Figure 3.1 - Recommended return periods for the determination of the characteristic values of climatic actions.

(a). A nominal duration of three days, to be chosen for short execution phases, corresponds to the extent in time of reliable meteorological predictions for the location of the site. This choice may be kept for a slightly longer execution phase if appropriate organizational measures are taken. The concept of mean return period is generally not appropriate for short term duration.

(b). For a nominal duration of up to three months actions may be determined taking into account appropriate seasonal and shorter term meteorological climatic variations. For example, the flood magnitude of a river depends on the period of the year under consideration.

A minimum wind velocity during execution may be defined in the National Annex or for the individual project. The recommended basic value for durations of up to 3 months is 20 m/s in accordance with EN 1991-1-4. Relationships between characteristic values and return period for climatic actions are given in the appropriate parts of EN 1991.

Note The rules for the combination of snow loads and wind actions with construction loads Q_c (see 4.11.1) should be defined. These rules may be defined in the National Annex or for the individual project. Actions due to wind excitation (including aerodynamic effects due to passing vehicles, including trains) that are likely to produce fatigue effects in structural members should be taken into account.

Actions due to creep and shrinkage in concrete construction works should be determined on the basis of the expected dates and duration associated with the design situations, where appropriate.

ULTIMATE LIMIT STATES. Ultimate limit states shall be verified for all selected transient, accidental and seismic design situations as appropriate during execution in accordance with EN 1990[(1)]. Generally, accidental design situations refer to exceptional conditions applicable to the structure or its exposure, such as impact, local failure and subsequent progressive collapse, fall of structural or non-structural parts, and, in the case of buildings, abnormal concentrations of building equipment and/or building materials, water accumulation on steel roofs, fire, etc.

SERVICEABILITY LIMIT STATES. The serviceability limit states for the selected design situations during execution shall be verified, as appropriate, in accordance with EN 1990. Operations during execution which can cause excessive cracking and/or early deflections and which may adversely affect the durability, fitness for use and/or aesthetic appearance in the final stage shall be avoided. Load effects due to shrinkage and temperature should be taken into account in the design and should be minimized by appropriate detailing.

The combinations of actions should be established in accordance with EN 1990:2002, 6.5.3 (2). In general, the relevant combinations of actions for transient design situations during execution are:

— the characteristic combination

— the quasi-permanent combination.

1.3 Representation of main actions

Characteristic and other representative values of actions shall be determined in accordance with EN 1990, EN 1991, EN 1997 and EN 1998.

Note The representative values of actions during execution may be different from those used in the design of the completed structure.

Representative values of construction loads (Q_c) should be determined taking into account their variations in time. The representative values of actions during execution may be different from those used in the design of the completed structure.

(1) See also EN 1991-1-7.

Common actions during execution, specific construction loads and methods for establishing their values are the following:

— actions on structural and non-structural members during handling
— geotechnical actions
— actions due to prestressing
— effects of pre-deformations
— temperature, shrinkage, hydration effects
— wind actions
— snow loads
— actions caused by water
— actions due to atmospheric icing
— construction loads.

ACTIONS ON STRUCTURAL AND NON-STRUCTURAL MEMBERS DURING HANDLING. The self-weight of structural and non-structural members during handling should be determined in accordance with EN 1991-1-1. Dynamic or inertia effects of self-weight of structural and non-structural members should be taken into account.

GEOTECHNICAL ACTIONS. The characteristic values of geotechnical parameters, soil and earth pressures, and limiting values for movements of foundations shall be determined according to EN 1997.

ACTIONS DUE TO PRESTRESSING. Loads on the structure from stressing jacks during the prestressing activities should be classified as variable actions for the design of the anchor region. Prestressing forces during the execution stage should be taken into account as permanent actions.

EFFECTS OF PRE-DEFORMATIONS. The treatment of the effects of pre-deformations shall be in conformity with the relevant design Eurocode (from EN 1992 to EN 1999). The action effects from pre-deformations should be checked against design criteria by measuring forces and deformations during execution.

TEMPERATURE, SHRINKAGE, HYDRATION EFFECTS. The effects of temperature, shrinkage and hydration shall be taken into account in each construction phase, as appropriate. For buildings, the actions due to temperature and shrinkage are not generally significant if appropriate detailing has been provided for the persistent design situation. In the case of bridges, for the determination of restraints to temperature effects of friction at bearings, that permit free movements, they should be taken into account on the basis of appropriate representative values.(1)

WIND ACTIONS. The need for a dynamic response design procedure for wind actions should be determined for the execution stages, taking into account the degree of completeness and stability of the structure and its various elements. Where a dynamic response procedure is not needed, the characteristic values of static wind forces Q_W should be determined according to EN 1991-1-4 for the appropriate return period.

(1) See EN 1337.

The effects of wind induced vibrations such as vortex induced cross wind vibrations, galloping flutter and rainwind should be taken into account, including the potential for fatigue of, for example, slender elements. When determining wind forces, the areas of equipment, falsework and other auxiliary construction works that are loaded should be taken into account.

SNOW LOADS. Snow loads shall be determined according to EN 1991-1-3 for the conditions of site and the required return period.(1)

ACTIONS CAUSED BY WATER. In general, actions due to water, including ground water, (Q_{wa}) should be represented as static pressures and/or hydrodynamic effects, whichever gives the most unfavourable effects. Actions caused by water may be taken into account in combinations as permanent or variable actions.

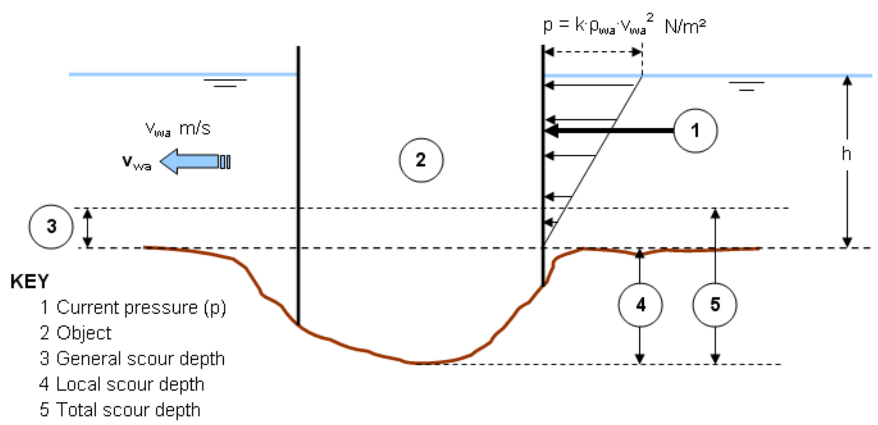

Figure 1.33 From Figure 4.1 - Pressure and force due to currents.

The magnitude of the total horizontal force F_{wa} (N) exerted by currents on the vertical surface should be determined by expression 4.1. See also Figure 4.1:

$$F_{wa} = \frac{1}{2}k\rho_{wa}hbv_{wa}^2 \qquad \text{(Eq. 1-48)}$$

where:

- v_{wa} is the mean speed (m/s) of the water averaged over the depth "h"
- h is the water depth (m), but not including local scour depth
- b is the width (m) of the object
- ρ_{wa} is the density of water (kg/m^3)
- k is the shape factor: $k = 1,44$ for an object of square or rectangular horizontal cross-section, and $k = 0,70$ for an object of circular horizontal cross-section.

(1) For bridges see also Annex A2.

F_{wa} may be used to check the stability of bridge piers and cofferdams, etc. A more refined formulation may be used to determine F_{wa} for the individual project. The effect of scour may be taken into account for the design where relevant. See 3.1(12) and 1.5.2.3 and 1.5.2.4.

Where relevant, the possible accumulation of debris should be represented by a force F_{deb} (N) and calculated for a rectangular object (e.g. cofferdam), for example, from:

$$F_{deb} = k_{deb} A_{deb} v_{wa}^2 \qquad \textbf{(Eq. 1-49)}$$

where:

- k_{deb} is the debris density (kg/m^3) parameter
- v_{wa} is the mean speed (m/s) of the water averaged over the depth
- A_{deb} is the area (m^2) of obstruction presented by the trapped debris and falsework.

Expression above may be adjusted for the individual project, taking account of its specific environmental conditions. The recommended value of k_{deb} is 666 kg/m^3.

ACTIONS DUE TO ATMOSPHERIC ICING. The representative values of these actions may be defined in the National Annex or for the individual project. Guidance may be found in EN 1993-3 and in ISO 12494.

CONSTRUCTION LOADS. Construction loads (Q_c) may be represented in the appropriate design situations (see EN 1990), either, as one single variable action, or where appropriate different types of construction loads may be grouped and applied as a single variable action. Single and/or a grouping of construction loads should be considered to act simultaneously with non-construction loads as appropriate. Usually, they are modelled as free actions. Construction loads to be included for consideration are given in Table 4.1 (“*Representation of construction loads* Q_c”). They are the following:

1. Q_{ca} personnel and hand tools (working personnel, staff and visitors with hand tools or other small site equipment)
2. Q_{cb} storage of movable items (building and construction materials, precast elements, equipment)
3. Q_{cc} non-permanent equipment in position for use during execution (formwork panels, scaffolding, falsework, machinery, containers) or during movement (travelling forms, launching girders and nose, counterweights)
4. Q_{cd} movable heavy machinery and equipment (cranes, lifts, vehicles, power installations, jacks, heavy lifting devices and trucks)
5. Q_{ce} accumulation of waste materials (surplus of construction materials or excavated soil, demolition materials)
6. Q_{cf} loads from part of structure in a temporary state or loads from lifting operations.

Construction loads Q_c may be represented in the appropriate design situations (see EN 1990), either, as one single variable action, or where relevant by a group of different types of construction loads, which is applied as a single variable action. Single and/or a grouping of construction loads should be considered to act simultaneously with Non construction loads as appropriate.

Recommended values of ψ factors for construction loads are given in Annex A1 of this standard for buildings, and in Annex A2 to EN 1990 for bridges.

1.4 Construction loads during the casting of concrete

Actions to be taken into account simultaneously during the casting of concrete may include working personnel with small site equipment (Q_{ca}), formwork and load-bearing members (Q_{cc}) and the weight of fresh concrete (which is one example of Q_{cf}), as appropriate.

For the density of fresh concrete see EN 1991-1-1:2002 Table A.1. Q_{ca}, Q_{cc} and Q_{cf} may be given in the National Annex. Recommended values of actions due to construction loads during casting of concrete (Q_{cf}) may be taken from Table 4.2, and for fresh concrete from EN 1991-1-1;2002, Table A.1. Other values may have to be defined, for example, when using self-levelling concrete or precast products. Loads according to (1), (2) and (3), as given in Table 4.2, are intended to be positioned to cause the maximum effects, which may be symmetrical or not.

Action	Loaded area	Load [kN/m^2]
(1)	Outside the working area	0,75 (covering Q_{ca})
(2)	Inside the working area (3 m) x (3 m) - or the span length if less	$0,75 \leq 0,10Q_{cf} \leq 1,50$ (includes Q_{ca} and Q_{cf})
(3)	Actual area	Self-weight of the formwork, load-bearing element (Q_{cc}) and the weight of the fresh concrete for the design thickness (Q_{cf})

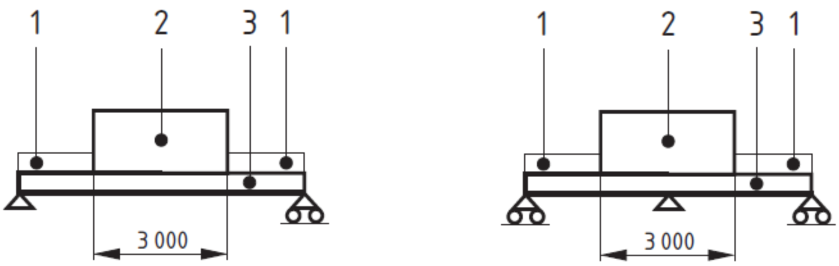

Table 1.16 From Table 4.2 - Recommended characteristic values of actions due to construction loads during casting of concrete.

1.5 Accidental actions

Accidental actions such as impact from construction vehicles, cranes, building equipment or materials in transit (e.g. skip of fresh concrete), and/or local failure of final or temporary supports, including dynamic effects, that may result in collapse of load-bearing structural members, shall be taken into account, where relevant.

The effects of the accidental actions should be assessed to determine the potential for inducing movement in the structure, and also the extent and effect of any such movement should be determined, with the potential for progressive collapse assessed.

1.6 Seismic actions

Seismic actions should be determined according to EN 1998, taking into account the reference period of the considered transient situation.

1.7 Verification tests

EN1991-1-6.XLS. 12.47 MB. Created: 12 December 2013. Last/Rel.-date: 12 December 2013. Sheets:

- Splash
- CodeSec2to4
- BridgeDeck.

EXAMPLE 1-Z- Return period for climatic action during execution - **test1**

Given: According to Table 3.1 "*Recommended return periods for the determination of the characteristic values of climatic actions*" find the characteristic value of the snow load on the ground for a nominal duration of 14 days. Let us assume a characteristic snow load on the ground (with a return period of 50 years) $s_k = 0,60\ \text{kN/m}^2$. Suppose that the available data show that the annual maximum snow load can be assumed to follow a Gumbel probability distribution with a coefficient of variation of annual maximum snow loads equal to 0,3.

[*Reference sheet:* CodeSec2to4]-[*Cell-Range: A67:O67-A104:O104*].

Solution: Enter Duration column Table 3.1:

$\leq$ *3 months (but > 3 days)*, with a return period T = 5 years

we can find the annual probability of exceedence: $p \approx \frac{1}{T} = \frac{1}{5} = 0,20$.

The relationship between the characteristic value of the snow load on the ground and the snow load on the ground for a mean recurrence interval of n years is given by [Annex D of EN 1991-1-3]:

$$\frac{s_n}{s_k} = \frac{1 - V\frac{\sqrt{6}}{\pi}\ln[-\ln(1-p)] + 0,57722}{(1 + 2,5923V)} = \frac{1 - 0,3\frac{\sqrt{6}}{\pi}\ln[-\ln(1-0,20)] + 0,57722}{(1 + 2,5923 \cdot 0,3)} \approx 0,69$$

$s_n = 0,69 \cdot (0,60\ kN/m^2) = 0,41\ kN/m^2$ (rounded value).

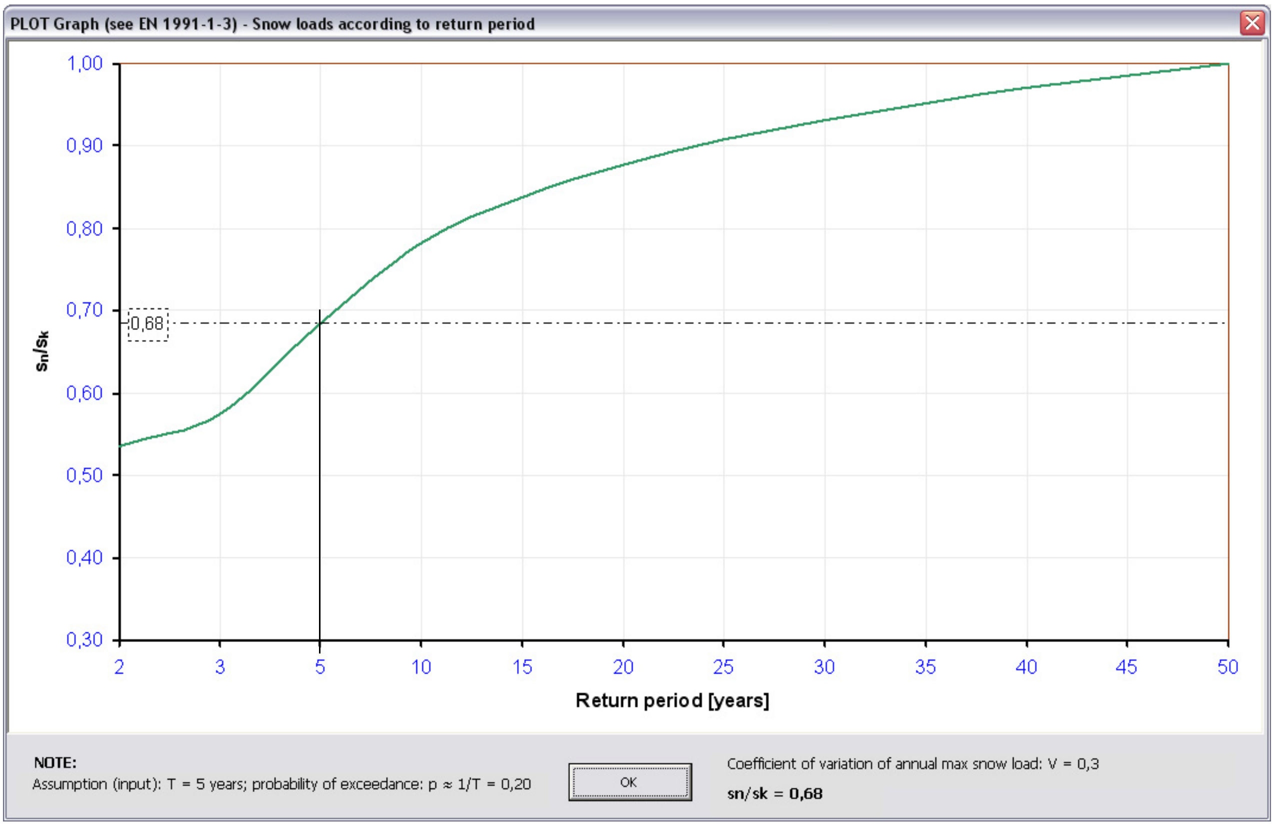

Figure 1.34 Snow loads according to return period (Excel output).

Relationships between characteristic values and return period for climatic actions are given in the appropriate Parts of EN 1991.

example-end

EXAMPLE 1-AA- Actions caused by water - **test2**

Given: Determine the magnitude of the total horizontal force F_{wa} exerted by a river currents on the vertical surface of a bridge pier whose width perpendicularly to the water speed is 4 meters long.

Let us assume a slender pier with a cross-section of square shape (4 m x 4 m), a water depth h = 4 m (not including local scour depth) and an averaged mean water speed $v_{wa} = 0,95$ m/s. Calculate the force on the slender pier due to a possible accumulation of debris as well.

[*Reference sheet:* CodeSec2to4]-[*Cell-Range: A275:O275-A344:O344*].

Solution: According to Eurocodes for an object of rectangular horizontal cross-section the shape factor is equal to 1,44 regardless of the value of the Reynolds number. In this case, with $b = L = 4$ m the Reynolds number is (@ 1 Atm):

$$\mathrm{Re} = \frac{v_{wa}L}{\nu} = \frac{(0,95\ \mathrm{m/s})\cdot(4\ \mathrm{m})}{(1,52\times10^{-6}\mathrm{m^2s^{-1}})} = 2,50\times10^{6}\ [-].$$

According to some scientific publications(1) is:

$k = 2,0$ for $2\times10^4 \le \mathrm{Re} \le 2,50\times10^6$. Therefore:

$$F_{wa} = \frac{1}{2}k\rho_{wa}hbv_{wa}^2 = \frac{1}{2}(2,0)(1000)(4)(4)(0,95)^2 = 14440\ \mathrm{N} = 14,44\ \mathrm{kN}.$$

$$F_{wa}/b = (14,44\ \mathrm{kN})/(4\ \mathrm{m}) = 3,61\ \mathrm{kN/m}.$$

Assuming (say) $A_{deb} = b\cdot(1,50\ \mathrm{m}) = 4\cdot1,50 = 6\ \mathrm{m}^2$ we get:

$$F_{deb} = k_{deb}A_{deb}v_{wa}^2 = (666)\cdot(6)\cdot(0,95)^2 = 3606,4\ \mathrm{N} = 3,61\ \mathrm{kN}.$$

example-end

EXAMPLE 1-AB- Action during execution on bridge slabs - **test3**

Given: A continuous composite deck of a road bridge is made up of two steel girders with I cross-section and a concrete slab with total width 12,0 m. The slab depth, with a 2.5% symmetrical superelevation, varies from 0,4 m over the girders to 0,25 m at its free edges and 0,30 m at the central point. The centre-to-centre spacing between main girders is 7 m and the slab cantilever either side is 2,5 m long.(2) Find at least two different load cases that could be envisaged in principle to maximize effects on the slab cross sections on the support and on the midspan, respectively. Consider a fresh concrete weight Q_{cf} of about 7,50 kN/m² in this example.

[*Reference sheet:* BridgeDeck]-[*Cell-Range: A275:O275-A344:O344*].

Solution: During the casting of the concrete slab, working personnel (Q_{ca}), formwork and load-bearing members (Q_{cc}) and weight of the fresh concrete, which is classified as Q_{cf}, should be considered acting simultaneously.

(1) "Meccanica dei Fluidi", Marchi E., Rubatta A. - UTET, 1981.

(2) Reference to the design of a three span continuous steel-concrete composite two girders bridge. JRC Scientific and Technical Reports. Bridge Design to Eurocodes Worked examples. Worked examples presented at the Workshop "Bridge Design to Eurocodes", Vienna, 4-6 October 2010. Support to the implementation, harmonization and further development of the Eurocodes. Editors A. Athanasopoulou, M. Poljansek, A. Pinto G., Tsionis, S. Denton.

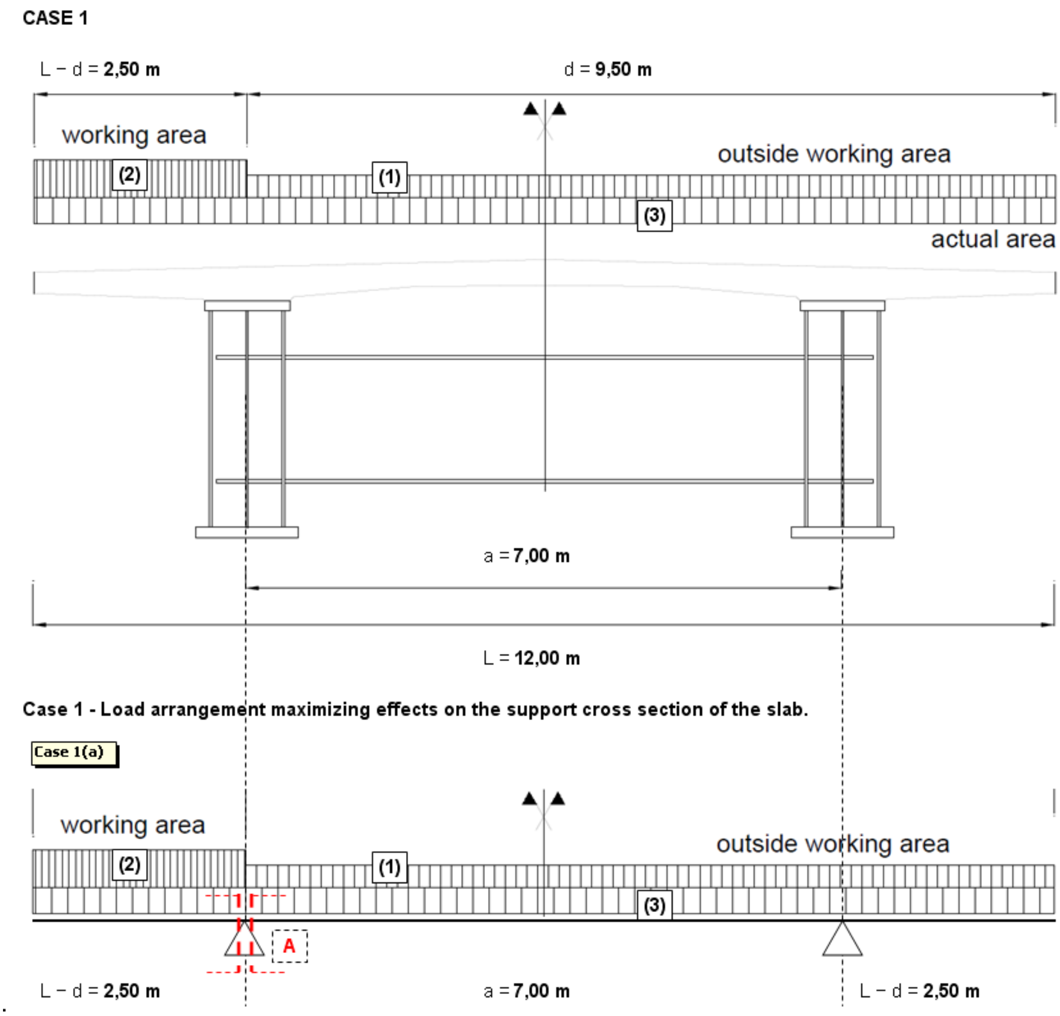

Figure 1.35 Load arrangement maximizing effects on the support cross section of the slab.

According to EN 1991-1-7 recommendations, during the concrete casting of the deck, in the actual area it can be identified two parts, the working area, which is a square whose side is the minimum between 3,0 m and the span length, and the remaining (outside the working area).

The actual area is loaded by the self-weight of the formwork and load bearing element Q_{cc} and by the weight of the fresh concrete Q_{cf}, the working area by $0,10Q_{cf}$, with the restriction $0,75\ kN/m^2 \leq 0,10 \cdot Q_{fc} \leq 1,50\ kN/m^2$, and the area outside the working area by $0,75\ kN/m^2$, covering Q_{ca}.

According to Table 4.1 "*Representation of construction loads (Q_c)*" and figure in Table 4.2 "*Recommended characteristic values of actions due to construction loads during casting of concrete*" we have (see figure above):

(1) Load action (outside working area):

$$Q_{ca} = 1,00\ kN/m^2$$

(2) Load action (working area)

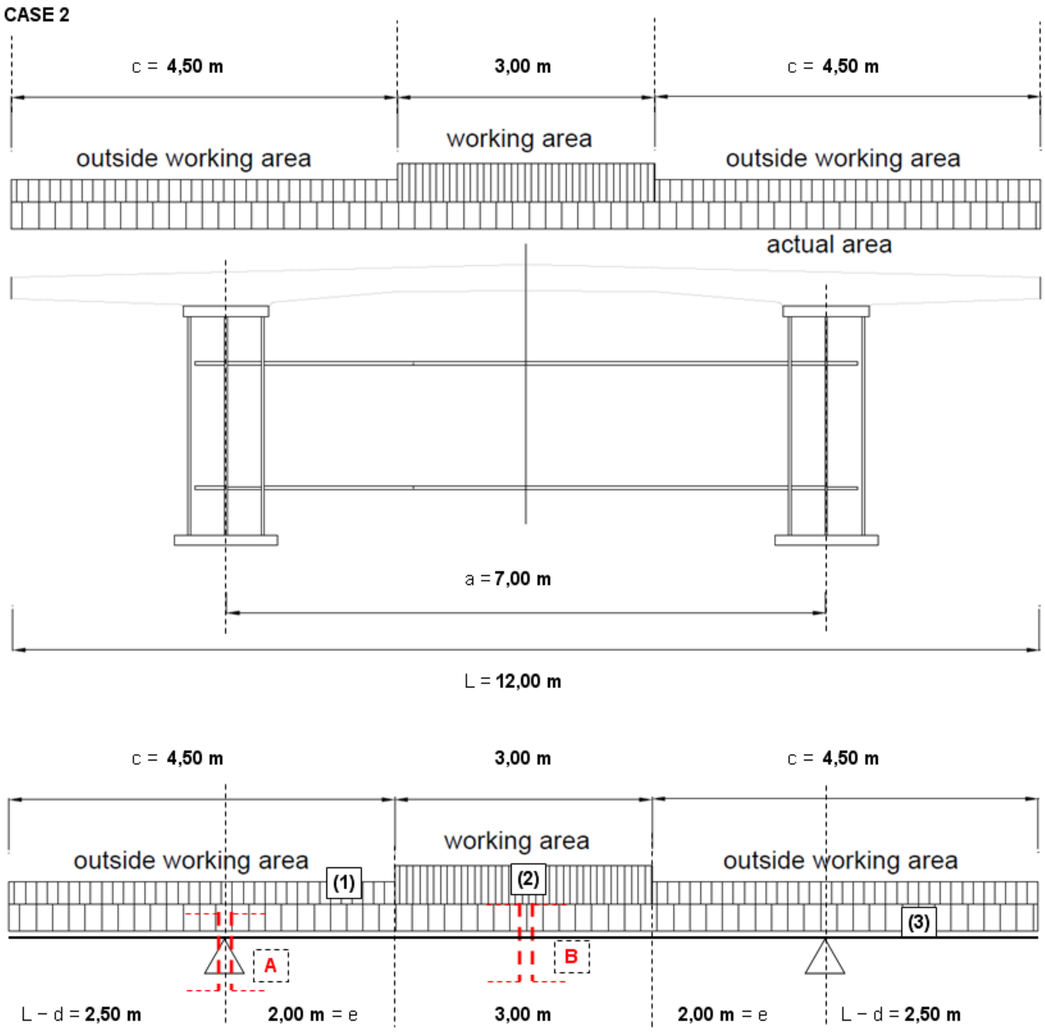

Figure 1.36 Load arrangement maximizing effects on the midspan of the slab.

$$Q_{ca} + Q_{cf} = (1,00 + 7,50) = 8,50 \text{ kN/m}^2$$

(3) Load action (actual area)

$$Q_{cc} + Q_{cf} = (0,50 + 7,50) = 8,00 \text{ kN/m}^2 .$$

From which, using the given numerical data, we get:

CASE 1

Support A (SX) - shear forces (characteristic values):

$V_{kA(SX)} = 0$ for load action **(1)**

$V_{kA(SX)} = -(Q_{ca} + Q_{cf}) \cdot (L - d) = -(8,50) \cdot (2,50) = -21,25 \text{ kN/m}$ for load action **(2)**

$V_{kA(SX)} = -(Q_{cc} + Q_{cf}) \cdot (L - d) = -(8,00) \cdot (2,50) = -20,00 \text{ kN/m}$ for load action **(3)**.

Support A (SX) - Bending moments (characteristic values):

$M_{kA} = 0$ for load action **(1)**

$M_{kA} = -0,5 \cdot (Q_{ca} + Q_{cf}) \cdot (L-d)^2 = -0,5 \cdot (8,50) \cdot (2,50)^2 = -26,56\ kNm/m$ for load action **(2)**

$M_{kA} = -0,5 \cdot (Q_{cc} + Q_{cf}) \cdot (L-d)^2 = -0,5 \cdot (8,00) \cdot (2,50)^2 = -25,00\ kNm/m$.

CASE 2

Support A (DX) - shear forces (characteristic values):

$V_{kA(DX)} = Q_{ca} \cdot e = (1,00) \cdot (2,00) = 2,00\ kN/m$ for load action **(1)**.

Mid-span B - Bending moments (characteristic value):

$M_{kB} = 0,5Q_{ca}e^2 - 0,5Q_{ca} \cdot (L-d)^2 = 0,5(1,00)(2,00)^2 - 0,5(1,00) \cdot (2,50)^2 = -1,13\ kNm/m$

for load action **(1)**.

Support A (DX) - shear forces (characteristic values):

$V_{kA(DX)} = 0,5 \cdot (Q_{ca} + Q_{cf}) \cdot 3 = 0,5 \cdot (8,50) \cdot 3 = 12,75\ kN/m$ for load action **(2)**.

Mid-span B - Bending moments (characteristic value):

$$M_{kB} = \frac{(Q_{ca} + Q_{cf}) \cdot 3 \cdot (2a-3)}{8} = \frac{(8,50) \cdot 3 \cdot [2 \cdot (7) - 3]}{8} = 35,06\ kNm/m$$

for load action **(2)**.

Support A (DX) - shear forces (characteristic values):

$V_{kA(DX)} = 0,5 \cdot (Q_{cc} + Q_{cf}) \cdot [a + 2(L-d)] - (Q_{cc} + Q_{cf}) \cdot (L-d)$

$V_{kA(DX)} = 0,5 \cdot (8,00) \cdot [7,00 + 2(2,50)] - (8,00) \cdot (2,50) = 28,00\ kN/m$ for load action **(3)**.

Mid-span B - Bending moments (characteristic value):

$$M_{kB} = \frac{(Q_{cc} + Q_{cf}) \cdot [a^2 - 4(L-d)^2]}{8} = \frac{(8,00) \cdot [(7,00)^2 - 4(2,50)^2]}{8} = 24,00\ kNm/m$$

for load action **(3)**.

Design load (ULS)

With all partial safety factors equal to 1,5 the ultimate design loads are the following:

CASE 1 (@ support A)

shear force: $V_{Ed} = 1,50 \cdot (0 - 20,00 - 21,25) = -61,88\ kN/m$

bending moment: $M_{Ed} = 1,50 \cdot (0 - 25,00 - 26,56) = -77,34\ kNm/m$.

CASE 2 (@ support A)

shear force: $V_{Ed} = 1,50 \cdot (2,00 + 28,00 + 12,75) = 64,13\ kN/m$.

Bending moment **(@ mid-span)**:

$M_{Ed} = 1,50 \cdot (-1,13 + 24,00 + 35,06) = 86,90\ kNm/m$.

Note. Loads Q_{ce} due to accumulation of waste materials may vary significantly, and over short time periods, depending on types of materials, climatic conditions, build-up and clearance rates, and they can also induce possible mass effects on horizontal, inclined and vertical elements (such as walls).

example-end

EXAMPLE 1-AC- Pre-dimensioning and calculation of the bridge slab transverse reinforcing steel - **test4**

Given: Let us suppose that the design moment over the main steel girdes and at mid-span of the slab (between the main steel girders) is equal to approximately 3 times the maximum value calculated during execution phase. The slab depth varies from 0,4 m over the girders to 0,25 m at its free edges and 0,30 m at the central point. Find the main tensile transverse reinforcement, using for the concrete a simplified rectangular stress distribution (EN 1992-1-1 Cl. 3.1.7(3)) for grades of concrete up to C50/50. (Concrete class C35/45 and reinforcing bars used are class B high bond bars with a yield strength $f_{yk} = 500\ \text{MPa}$.

[*Reference sheet:* BridgeDeck]-[PreCalculus Excel® form: *Cell-Row: 133 and 241*].

Solution: For grades of concrete up to C50/60:

$f_{cd} = \alpha_{cc} f_{ck}/\gamma_c = 0,85 f_{ck}/1,5$

$f_{yd} = f_{yk}/\gamma_s = f_{yk}/1,15 = 0,87 f_{yk}$.

For singly reinforced sections, the design equations can be derived as follows:

$F_c = (0,85 f_{ck}/1,5) \cdot b \cdot (0,8x) = 0,4533 f_{ck} bx$ (compression concrete)

$F_{st} = (0,87 f_{yk}) \cdot A_s$ (main tensile steel reinforcement).

Design moment about the centre of the tension force (beam lever arm "z"):

$M = (0,4533 f_{ck} bx) \cdot z = (0,4533 f_{ck} b) \cdot [2,5(d-z)] \cdot z = 1,1333 f_{ck} b(zd - z^2)$.

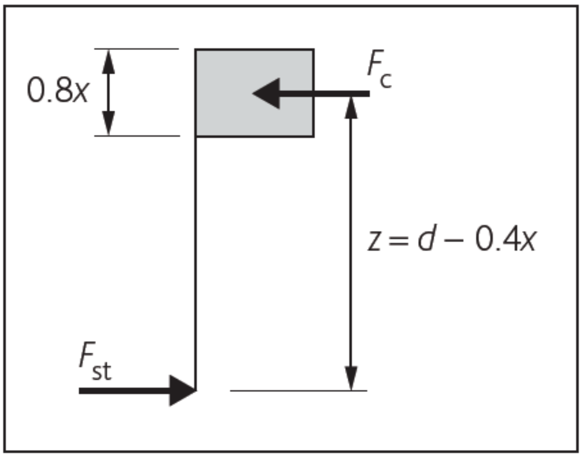

Figure 1.37 Singly reinforced section: beam lever arm "z".

Let $K = M/(bd^2 f_{ck})$, therefore

$$K = \frac{M}{bd^2 f_{ck}} = \frac{1,1333 f_{ck} b(zd - z^2)}{bd^2 f_{ck}} = 1,1333\left[\frac{z}{d} - \left(\frac{z}{d}\right)^2\right].$$

Solving the quadratic equation with the limit $d > 0,8x$:

$$\frac{z}{d} = 0,5 \cdot [1 + \sqrt{1 - (3,529K)}], \; z = 0,5d \cdot [1 + \sqrt{1 - (3,529K)}].$$

Note It is considered good practice in the UK to limit "z" to the maximum value 0,95d. This guards against relying on very thin sections of concrete which at the extreme top of a section may be of questionable strength.

Taking moments about the centre of the compression force we obtain the area of the main tensile reinforcement:

$$A_s = \frac{M}{(0,87 f_{yk}) \cdot z} \text{ (req' d)}$$

if $K < K'$, where K' is used to limit the depth of the neutral axis to avoid "overreinforcement" (i.e. to ensure that the reinforcement is yelding at failure, thus avoiding brittle failure of the concrete). Conversely, if $K > K'$ the section should be resized or compression reinforcement is required.

Note In line with consideration of good practice outlined above, the Excel form (Bending) allows the free choice of the redistribution ratio δ with the following recommended values:(1) $k_1 = 0{,}44$, $k_2 = 1{,}25(0{,}6 + 0{,}0014/\varepsilon_{cue})$ and $k_5 = 0{,}7$ (class B and C steel reinforcement).

For example, to obtain $K' = 0{,}167$ the input requires $\delta = 1$ (default input value).

Design ultimate bending moment due to ultimate loads:

$$M_{Ed} \approx 3 \cdot \max[M_{kA};M_{kB}] = 3 \cdot (87 \text{ kNm/m}) \approx 260 \text{ kNm/m}.$$

Let us assume an effective depth of tension reinforcement of about:

$d = 0,85h = 0,85 \cdot (300 \text{ mm}) = 255 \text{ mm}$ (at mid-span of the slab)

$d = 0,85h = 0,85 \cdot (400 \text{ mm}) = 340 \text{ mm}$ (section above the main steel girders).

At mid-span:

$$K = \frac{M_{Ed}}{(bd^2 f_{ck})} = \frac{260 \cdot 10^6}{(1000)(255)^2(35)} = 0,114 < K' = 0,167 \text{ (singly reinforced section).}$$

Section above the main steel girders:

$$K = \frac{M_{Ed}}{(bd^2 f_{ck})} = \frac{260 \cdot 10^6}{(1000)(340)^2(35)} = 0,064 < K' = 0,167 \text{ (singly reinforced section).}$$

(1) EN 1992-1-1 Cl. 5.5(4) - Note.

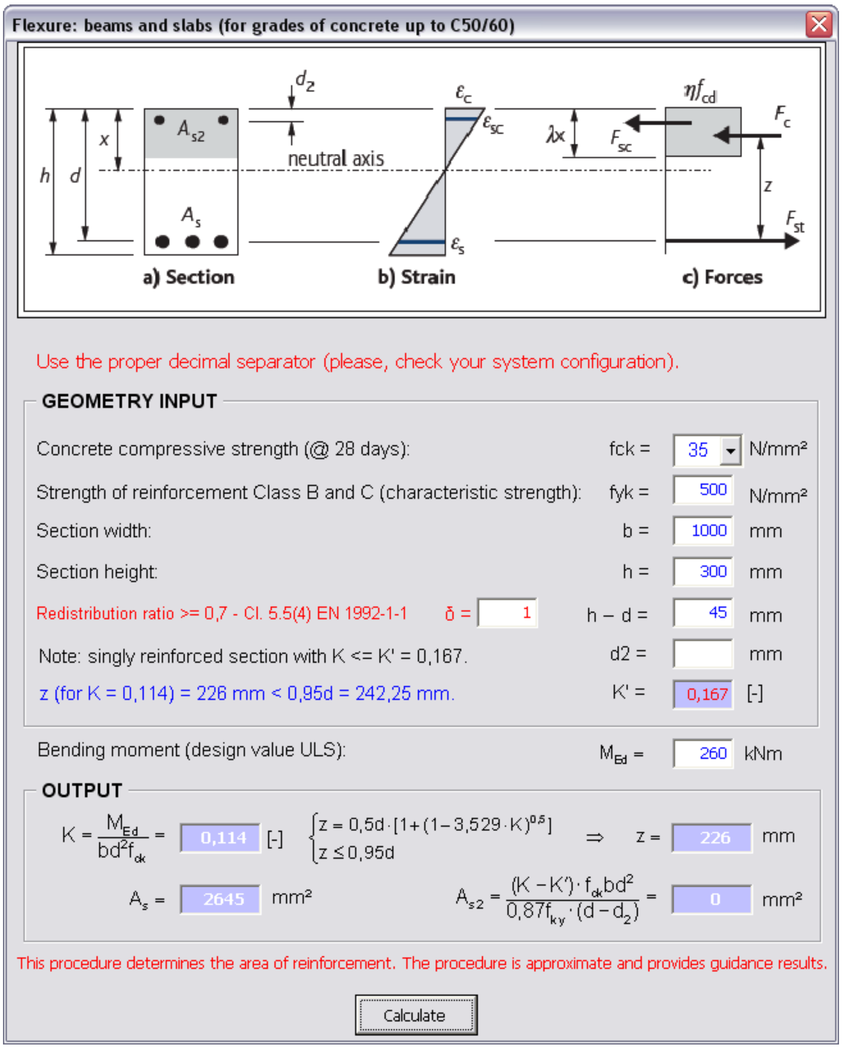

Figure 1.38 PreCalculus Excel® form: procedure for a quick pre-calculation: @ mid-span.

At mid-span:

$$z = 0,5d \cdot [1 + \sqrt{1 - (3,529K)}] = 0,5(255) \cdot [1 + \sqrt{1 - [3,529(0,114)]}] = 226,1 \text{ mm} < 0,95d$$

Section above the main steel girders:

$$z = 0,5d \cdot [1 + \sqrt{1 - (3,529K)}] = 0,5(340) \cdot [1 + \sqrt{1 - [3,529(0,064)]}] = 319,8 \text{ mm} < 0,95d.$$

Area of tensile reinforcement

At mid-span:

$$A_s = \frac{M_{Ed}}{(0,87f_{yk}) \cdot z} = \frac{260 \cdot 10^6}{(0,87 \cdot 500) \cdot (226)} = 2644 \text{ mm}^2/\text{m (try H25 @ 170 mm)}$$

Section above the main steel girders:

$$A_s = \frac{M_{Ed}}{(0,87f_{yk}) \cdot z} = \frac{260 \cdot 10^6}{(0,87 \cdot 500) \cdot (320)} = 1869 \text{ mm}^2/\text{m (try H20 @ 170 mm).}$$

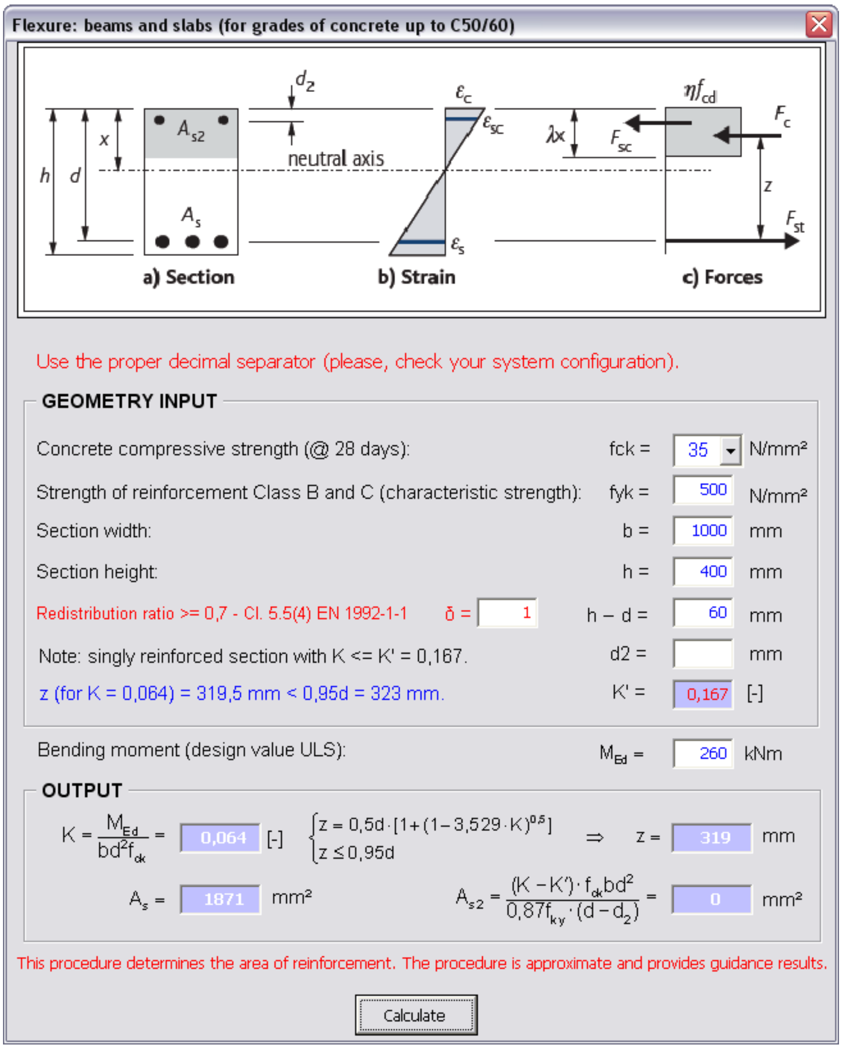

Figure 1.39 PreCalculus Excel® form: procedure for a quick pre-calculation: @ section above the main steel girders.

1.8 References [Section 1]

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1.9 Vba References

MODULE NAME: modPastePicture. Author: STEPHEN BULLEN, Office Automation Ltd - 15 November 1998. http://www.oaltd.co.uk.