Section 1 Eurocode 0 - EN1990

1.1 Foreword

EN 1990 "*Eurocode: Basis of structural design*" is the head document in the Eurocode suite. It describes the basis and general principles for the structural design and verification of buildings and civil engineering works including geotechnical aspects, the principles and requirements for safety and serviceability of structures and guidelines for related aspects of structural reliability in all circumstances in which a structure is required to give adequate performance, including fire and seismic events. Consisting of only one part, it is used with all the other Eurocodes (1 to 9) for design.

The Structural Eurocode programme comprises the following standards generally consisting of a number of Parts:

- EN 1990 Eurocode 0: Basis of Structural Design
- EN 1991 Eurocode 1: Actions on structures
- EN 1992 Eurocode 2: Design of concrete structures
- EN 1993 Eurocode 3: Design of steel structures
- EN 1994 Eurocode 4: Design of composite steel and concrete structures
- EN 1995 Eurocode 5: Design of timber structures
- EN 1996 Eurocode 6: Design of masonry structures
- EN 1997 Eurocode 7: Geotechnical design
- EN 1998 Eurocode 8: Design of structures for earthquake resistance
- EN 1999 Eurocode 9: Design of aluminium structures.

Eurocode standards recognise the responsibility of regulatory authorities in each Member State and have safeguarded their right to determine values related to regulatory safety matters at national level where these continue to vary from State to State.

1.2 National Standards implementing Eurocodes

The National Standards implementing Eurocodes will comprise the full text of the Eurocode (including any annexes), as published by CEN, which may be preceded by a National title page and National foreword, and may be followed by a National annex. The National annex may only contain information on those parameters which are left open in the Eurocode for national choice, known as Nationally Determined Parameters, to be used for the design of buildings and civil engineering works to be constructed in the country concerned, i.e.:

- values and/or classes where alternatives are given in the Eurocode
- values to be used where a symbol only is given in the Eurocode
- country specific data (geographical, climatic, etc.), e.g. snow map
- the procedure to be used where alternative procedures are given in the Eurocode.

It may also contain:

- decisions on the application of informative annexes
- references to non-contradictory complementary information to assist the user to apply the Eurocode.

1.3 National annex for EN 1990

This standard gives alternative procedures, values and recommendations for classes with notes indicating where national choices may have to be made. Hence the National Standard implementing EN 1990 should have a National annex containing all Nationally Determined Parameters to be used for the design of buildings and civil engineering.

1.4 Verification tests

EN1990.XLS. 4.4 MB. Created: 5 January 2013. Last/Rel.-date: 6 March 2013. Sheets:

— Splash
— Annex A1-B
— Annex C
— Annex D.

EXAMPLE 1-A- Target reliability index - **test 1**

Given: Target reliability index (1 year): $\beta_1 = 4,7$ (ultimate limit state: see tab. C2-EN1990). Find the probability of failure P_f (see ta. C1-EN1990) related to β and the value of β for a different reference period (say 100 years).

[*Reference sheet:* Annex C]-[*Cell-Range: A1:O1-A23:O23*].

Solution: In the Level II procedures (see Figure C1-EN1990 - *Overview of reliability methods*), an alternative measure of reliability is conventionally defined by the reliability index β which is related to the probably of failure P_f by:

$$P_f = \Phi(-\beta)$$

where Φ is the cumulative distribution function of the standardised Normal distribution.

The general formula for the probability density function of the Normal distribution is:

$$f(x) = \frac{\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]}{\sigma\sqrt{2\pi}} = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$$

where μ is the "location parameter" and σ is the "scale parameter". The case where $\mu = 0$ and $\sigma = 1$ is called the Standard Normal distribution. The equation for the standard normal distribution is:

$$f(x) = \frac{\exp\left[-\frac{x^2}{2}\right]}{\sqrt{2\pi}} = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}.$$

The probability of failure P_f can be expressed through a performance function g such that a structure is considered to survive if g > 0 and to fail if g ≤ 0: $P_f = \mathrm{Prob}(g \le 0)$.

If g is Normally distributed, β is taken as $\beta = \mu_g/\sigma_g$ (where μ_g is the mean value of g, and σ_g is the standard deviation), so that: $\mu_g - \beta\sigma_g = 0$ and $P_f = \mathrm{Prob}(g \le 0) = \mathrm{Prob}(g \le \mu_g - \beta\sigma_g)$.

The cumulative distribution function (CDF) $\Phi(-\beta)$ of a random variable is the probability of its value falling in the interval $[-\infty; \beta]$, as a function of x.

The CDF of the standard normal distribution, usually denoted with the capital Greek letter Φ, is the integral:

$$\Phi(\beta) = \int_{-\infty}^{\beta} f(x)dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{4,7} e^{-\frac{x^2}{2}} dx.$$

END NOTE

For $\beta = 4,7 = \beta_1$ (1 year):

$$\Phi(4,7) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{4,7} e^{-\frac{x^2}{2}} dx \approx 10^{-6}.$$

For a reference period of n = 100 years the reliability index β_n is (see eq. C.3-EN1990):

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n \quad \rightarrow \Phi(\beta_n) = [\Phi(4,7)]^{100} \Rightarrow [10^{-6}]^{100} = 10^{-4} \Rightarrow \beta_n = 3,7,$$

where $\Phi^{-1}(\beta_n) = \beta_n$ is the inverse of the cumulative distribution function. The quantile of the standard normal distribution is the inverse of the cumulative distribution function.

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example-end

EXAMPLE 1-B- Approach for calibration of design values (section C7-EN1990) - **test 2**

Given: Calculate the design values of action effects E_d and resistances R_d. Assume a target reliability index equal to $\beta = 4,8$. The standard deviations of the action effect and resistance are, respectively: $\sigma_E = 5,0$, $\sigma_R = 5,0$.

[*Reference sheet:* Annex C]-[*Cell-Range: A27:O27-A70:O70*].

Solution: The design values of action effects E_d and resistances R_d should be defined such that the probability of having a more unfavourable value is as follows [see (C.6a), (C.6b) EN1990]:

$$P(E > E_d) = \Phi(+\alpha_E \beta)$$

$$P(R \le R_d) = \Phi(-\alpha_R \beta).$$

substituting σ_E and σ_R into eq. (C.7)-EN1990, we obtain: $0,16 < \sigma_E / \sigma_R < 7,6$. The values of FORM sensivity factors α_E and α_R may be taken as $-0,7$ and $0,8$, respectively. This gives:

$$P(E > E_d) = \Phi(+\alpha_E\beta) = \Phi(-0,7\beta) = \Phi(-0,7 \times 4,8) = \Phi(-3,36) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-3,36} e^{-\frac{x^2}{2}} dx$$

$$P(R \le R_d) = \Phi(-\alpha_R\beta) = \Phi(-0,8\beta) = \Phi(-0,8 \times 4,8) = \Phi(-3,84) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-3,84} e^{-\frac{x^2}{2}} dx.$$

Using the given numerical data, we find (leading variable only):

$$P(E > E_d) = \Phi(+\alpha_E\beta) = 3,90 \times 10^{-4}$$

$$P(R \le R_d) = \Phi(-\alpha_R\beta) = 6,15 \times 10^{-5}.$$

When the action model contains several basic variables, for the accompanying actions the design value is defined by:

$$P(E > E_d) = \Phi(+\alpha_E 0,4\beta),$$

from which we obtain:

$$P(E > E_d) = \Phi(+\alpha_E 0,4\beta) = \Phi(-0,28\beta) = \Phi(-0,28 \times 4,8) = \Phi(-1,344) = 8,95 \times 10^{-2}.$$

example-end

EXAMPLE 1-C- Approach for calibration of design values (section C7-EN1990) - **test 3**

Given: Consider the same assumptions in the example above ($\beta = 4,8$). Assume $\sigma_E = 1,0$, $\sigma_R = 7,0$. Find the design values of action effects E_d and resistances R_d.

[*Reference sheet:* Annex C]-[*Cell-Range: A27:O27-A70:O70*].

Solution: The condition $0,16 < \sigma_E / \sigma_R < 7,6$ is not satisfied: $\alpha = \pm 1,0$ should be used for the variable with the larger standard deviation, and $\alpha = \pm 0,4$ for the variable with the smaller standard deviation.

The value of α is negative for unfavourable actions and action effects, and positive for resistances. Using these values of α, the design equations become:

$$P(E > E_d) = \Phi(+\alpha_E \beta) = \Phi(-0,4\beta) = \Phi(-0,4 \times 4,8) = 2,74 \times 10^{-2}$$

$$P(R \le R_d) = \Phi(-\alpha_R \beta) = \Phi(-1,0\beta) = \Phi(-4,8) = 7,93 \times 10^{-7}.$$

For the accompanying actions the design value is (smaller standard deviation):

$$P(E > E_d) = \Phi(+\alpha_E 0,4\beta) = \Phi(-0,4 \cdot 0,4\beta) = \Phi(-0,16 \times 4,8) = 2,21 \times 10^{-1}.$$

example-end

EXAMPLE 1-D- Approach for calibration of design values (section C7-EN1990) - **test 4a**

Given: Derive the design values of variables with a probability equal to 10^{-4} (reliability index around $\beta = 3,8$) using a Gumbel distribution. Assume:

$\mu_E = 30$; $\sigma_E = 1,0$

$\mu_R = 30$; $\sigma_R = 7,0$

(mean value and standard deviation of the action effect and resistance, respectively).

[*Reference sheet:* Annex C]-[*Cell-Range: A74:O74-A140:O140*].

Solution: Considering the same assumptions in the example above (condition $0,16 < \sigma_E / \sigma_R < 7,6$ not satisfied), it is seen that:

α_E	$-\alpha_E \beta$	σ_E / μ_E	$\Phi(-\alpha_E \beta)$	α_R	$-\alpha_R \beta$	σ_R / μ_R	$\Phi(-\alpha_R \beta)$
- 0,40	1,52	0,033	9,36 x 10^{-1}	1,00	- 3,80	0,233	7,23 x 10^{-5}

Table 1.1 Input data. See previous examples.

From table C3-EN1990 - *Design values for various distribution functions*, by using the Gumbel distribution with the given numerical data, it follows that:

$$a = \frac{\pi}{\sigma_R \sqrt{6}} = \frac{\pi}{7\sqrt{6}} = 0,183 \; ; \; u = \mu_R - \frac{0,577}{a} = 30 - \frac{0,577}{0,183} = 26,85$$

$$a = \frac{\pi}{\sigma_E \sqrt{6}} = \frac{\pi}{1\sqrt{6}} = 1,283 \; ; \; u = \mu_E - \frac{0,577}{a} = 30 - \frac{0,577}{1,283} = 29,55 .$$

Therefore, it is (leading variable action):

$$X_{di,R} = u - \frac{1}{a} \cdot \ln\{-\ln[\Phi(-\alpha_R \beta)]\} = u - \frac{1}{0,183} \cdot \ln\{-\ln[7,235 \times 10^{-5}]\} = u - \frac{1}{0,183} \cdot 2,255$$

$$X_{di,\,R} = 26,85 - \frac{1}{0,183} \cdot 2,255 = 14,5$$

$$X_{di,\,E} = u - \frac{1}{a} \cdot \ln\{-\ln[\Phi(-\alpha_E\beta)]\} = u - \frac{1}{1,283} \cdot \ln\{-\ln[9,357 \times 10^{-1}]\} = u + \frac{1}{1,283} \cdot 2,712$$

$$X_{di,\,R} = 29,55 + \frac{1}{1,283} \cdot 2,712 = 31,7 .$$

example-end

EXAMPLE 1-E- Approach for calibration of design values (section C7-EN1990) - **test 4b**

Given: Considering the same assumptions in the example above (see tab. 1.1), derive the design values of action effects E_d with a probability equal to 10^{-4} using a Normal and a Log-normal distribution.

[*Reference sheet:* Annex C]-[*Cell-Range: A74:O74-A140:O140*].

Solution: Remembering that the condition $0,16 < \sigma_E/\sigma_R < 7,6$ is not satisfied, the design value of action effects for Normal distribution (see tab. C3-EN1990 - *Design values for various distribution functions*) becomes:

$$X_{di,\,E} = \mu - \alpha\beta\sigma = 30 + (0,40 \cdot 3,8 \cdot 1) = 31,5 .$$

Having calculated $V = \sigma_E/\mu_E = 1/30 = 0,033 < 0,2$, the design value of action effects for Log-normal distribution becomes:

$$X_{di,\,E} = \mu \cdot \exp(-\alpha\beta V) = 30 \cdot \exp(0,40 \cdot 3,8 \cdot 0,033) = 30 \cdot 1,051 = 31,6 .$$

example-end

EXAMPLE 1-F- Ψ_0 factors (section C10-EN1990) - **test 5**

Given: Use the expressions in tab. C4-EN1990 for obtaining the Ψ_0 factors in the case of two variable actions. Consider the following assumptions:

– reference period T = 50 years

– greater of the basic periods (for actions to be combined) $T_1 = 7$ years

– reliability index $\beta = 3,8$

– coefficient of variation V = 0,30 of the accompanying action (for the reference period).

[*Reference sheet:* Annex C]-[*Cell-Range: A144:O144-A189:O189*].

Solution: The distribution functions in Table C4 refer to the maxima within the reference period T. These distribution functions are total functions which consider the probability that an action value is zero during certain periods. The theory is based on the calculation of the inverse gamma distribution's probability density function of the extreme value of the accompanying action in the reference period.

The gamma distribution, like the Log-normal distribution, is a two-parameter family of continuous probability distributions. The general formula for the probability density function of the gamma distribution is:

$$f(x) = \frac{\left(\frac{x-\mu}{\beta}\right)^{\gamma-1}\exp\left(-\frac{x-\mu}{\beta}\right)}{\beta\Gamma(\gamma)};\ x \geq \mu;\ \gamma, \beta > 0.$$

where γ *is the shape parameter,* μ *is the location parameter,* β *is the scale parameter, and* $\Gamma(\gamma)$ *is the "gamma function which" has the formula:*

$$\Gamma(\gamma) = \int_0^{\infty} t^{\gamma-1}e^{-t}dt.$$

The case where $\mu = 0$ *and* $\beta = 1$ *is called the "standard gamma distribution". The equation for the standard gamma distribution reduces to:*

$$f(x) = \frac{x^{\gamma-1}e^{-x}}{\Gamma(\gamma)};\ x \geq 0;\ \gamma > 0.$$

The gamma distribution can be parameterized in terms of a shape parameter α *and an inverse scale parameter* $1/\gamma = \beta$*, called a rate parameter:*

$$g(x;\alpha,\beta) = \beta^{\alpha}\frac{1}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x},$$

With this parameterization, a $\mathrm{gamma}(\alpha, \beta)$ *distribution has mean* $\alpha\beta$ *and variance* $\alpha\beta^2$*. As in the log-normal distribution, x and the parameters* α *and* β *must be positive. The cumulative distribution function is the regularized gamma function:*

$$F(x) = P\{X \leq x\} = \int_0^{x} \beta^{\alpha}\frac{1}{\Gamma(\alpha)}t^{\alpha-1}e^{-\beta t}dt.$$

The inverse gamma distribution's probability density function is defined over the support x > 0:

$$[g(x;\alpha,\beta)]^{-1} = \frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{-\alpha-1}e^{-\beta/x}.$$

Therefore, the inverse $F^{-1}(x)$ *of the cumulative distribution function* $F(x)$ *is the quantile of the standard gamma distribution:* $F^{-1}(x) = x$.

END NOTE

Ratio approximated to the nearest integer: $N_1 = T/T_1 = 50/7 = 7,14 \rightarrow 7$.

Shape parameter α (gamma distribution):

$$k = (\mu/\sigma)^2 = (1/V)^2 = V^{-2} = (0,30)^{-2} = 11,1 = \alpha$$

Scale parameter β (gamma distribution):

$$\lambda = \frac{\mu}{\sigma^2} = \frac{\mu}{V^2\mu^2} = V^{-2} = (0,30)^{-2} = 11,1 \Rightarrow \beta = \frac{1}{\lambda} = (11,1)^{-1}$$

From table C4-EN1990:

$$\beta' = -\Phi^{-1}\left\{\frac{\Phi(-0,7\beta)}{N_1}\right\} = -\Phi^{-1}\left\{\frac{\Phi(-0,7\cdot 3,8)}{N_1}\right\} = -\Phi^{-1}\left\{\frac{\Phi(-2,66)}{7}\right\} = 3,3$$

$\Phi(0,4\beta') = \Phi(0,4\cdot 3,3) = \Phi(1,32) = 0,9066\,;\ [\Phi(0,4\beta')]^{N_1} = 0,9066^7 = 0,5034$

$\Phi(0,7\beta) = \Phi(0,7\cdot 3,8) = \Phi(2,66) = 0,9961\,;\ [\Phi(0,7\beta)]^{N_1} = 0,9961^7 = 0,9730\,;$

$-\ln(\Phi(0,7\beta)) = -\ln(0,9961) = 0,0039\,,$

$\Phi(-0,4\beta') = \Phi(-0,4\cdot 3,3) = \Phi(-1,32) = 0,0934$

$-N_1\Phi(-0,4\beta') = -7\cdot 0,0934 = -0,6538$

$\exp[-N_1\Phi(-0,4\beta')] = \exp[-0,6538] = 0,5200$

$\Phi(0,28\beta) = \Phi(0,28\cdot 3,8) = \Phi(1,06) = 0,8563\,;\ -\ln(\Phi(0,28\beta)) = -\ln(0,8563) = 0,1551$

Quantiles (for $\alpha = 11,1$, $\beta = 1/11,1$):

$F_S^{-1}\{[\Phi(0,4\beta')]^{N_1}\} = F_S^{-1}\{0,5034\} = 0,9727\,,\ F_S^{-1}\{[\Phi(0,7\beta)]^{N_1}\} = F_S^{-1}\{0,9730\} = 1,6546$

$F_S^{-1}\{\Phi(0,7\beta)\} = F_S^{-1}\{0,9961\} = 1,9797$

$F_S^{-1}\{\exp[-N_1\Phi(-0,4\beta')]\} = F_S^{-1}\{0,5200\} = 0,9850\,.$

Substituting the numerical data listed above into expressions in table C4-EN1990, we find:

a) General expression:

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{F_S^{-1}\{[\Phi(0,4\beta')]^{N_1}\}}{F_S^{-1}\{[\Phi(0,7\beta)]^{N_1}\}} = \frac{0,9727}{1,6546} = 0,588\,.$$

b) Approximation for very large N_1:

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{F_S^{-1}\{\exp[-N_1\Phi(-0,4\beta')]\}}{F_S^{-1}\{\Phi(0,7\beta)\}} = \frac{0,9850}{1,9797} = 0,497\,.$$

c) Normal (approximation):

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{1+(0,28\beta-0,7\ln N_1)V}{1+0,7\beta V} = \frac{1+(0,28\cdot 3,8-0,7\ln 7)(0,30)}{1+0,7\cdot(3,8\cdot 0,30)} = \frac{0,9106}{1,798} =$$

$$= 0,506\,.$$

d) Gumbel (approximation):

$$\Psi_0 = \frac{F_{accompanying}}{F_{leading}} = \frac{1-0,78V\{0,58+\ln[-\ln(\Phi(0,28\beta))]+\ln N_1\}}{1-0,78V\{0,58+\ln[-\ln(\Phi(0,7\beta))]\}} =$$

$$= \frac{1-0,78(0,30)\{0,58+\ln[0,1551]+\ln 7\}}{1-0,78(0,30)\{0,58+\ln[0,0039]\}} = \frac{1-0,78(0,30)\{0,58-1,8637+1,9459\}}{1-0,78(0,30)\{0,58-5,5468\}} =$$

$$= \frac{0,8450}{2,1622} = 0,391\,.$$

example-end

EXAMPLE 1-G- D7.2 Assessment via the characteristic value - **test 6**

Given: Find the design value of the property X considering already known the ratio η_d / γ_m between the design factor of the conversion factor and the partial factor of the material.

Suppose a simple random sample of size n = 30 is drawn from a population having mean μ and standard deviation σ (see table below). Suppose the original distribution is normal.

n	x_i	n	x_i
1	19,3	16	17,3
2	19,8	17	19,2
3	20,1	18	22,4
4	20,4	19	16,0
5	20,3	20	15,0
6	19,3	21	15,6
7	18,0	22	18,2
8	17,4	23	17,4
9	21,3	24	19,2
10	19,4	25	16,3
11	20,2	26	15,3
12	20,5	27	14,0
13	21,0	28	13,0
14	22,3	29	15,3
15	18,5	30	16,5

Table 1.2 Sample results (n = 30). Reference Sheet: Annex D. Cell-Range B50:B64 - E50:E64.

Find the mean, variance, standard deviation and the coefficient of variation of the sampling distribution. Rounding to the first decimal.

[*Reference sheet:* Annex D]-[*Cell-Range: A1:O1-A82:O82*].

Solution: Mean of the n = 30 sample results:

$$m_X = \frac{(19,3 + 19,8 + 20,1 + 20,4 + \ldots + 13,0 + 15,3 + 16,5)}{30} = 18,3\,.$$

Variance:

$$s_x^2 = \frac{1}{30-1}[(19,3 - 18,3)^2 + (19,8 - 18,3)^2 + (20,1 - 18,3)^2 + \ldots + (16,5 - 18,3)^2] = 6,0\,.$$

Standard deviation: $s_x = \sqrt{6,0} = 2,45$.

Coefficient of variation:

$$V_X = \frac{s_x}{m_x} = \frac{2,45}{18,3} = 0,13.$$

Values of k_n for the 5% characteristic value for n = 30 (see tab. D1-RN1990):

$$k_n = \begin{cases} 1,67 & V_x \text{ known} \\ 1,73 & V_x \text{ unknown} \end{cases}$$

Design value of the property X:

$$X_d = \eta_d \frac{X_{k(n)}}{\gamma_m} = \frac{\eta_d}{\gamma_m} m_x(1 - k_n V_x) = \frac{\eta_d}{\gamma_m} \cdot 18,3 \cdot \left(1 - \begin{bmatrix} 1,67 \\ 1,73 \end{bmatrix} 0,13\right) = \frac{\eta_d}{\gamma_m} \cdot \begin{cases} 14,3 & V_x \text{ known} \\ 14,2 & V_x \text{ unknown} \end{cases}$$

having considered already known the ratio η_d / γ_m.

example-end

EXAMPLE 1-H- D7.2 Assessment via the characteristic value - **test 7**

Given: Considering the same sample result in the example above (see tab. 1.2) and supposing the original distribution is Log-normal, find the design value of a property X considering already known the ratio η_d / γ_m. Rounding to the first decimal.

[*Reference sheet:* Annex D]-[*Cell-Range: A84:O84-A125:O125*].

Solution: Estimated value m_y for $E(\Delta)$:

$$m_y = \bar{\Delta} = \frac{1}{n}\sum_{i=1}^{n} \ln(\delta_i) = \frac{1}{n}\sum_{i=1}^{n} \Delta_i = \frac{1}{30}[\ln(19,3) + \ln(19,8) + \ldots + \ln(15,3) + \ln(16,5)] = 2,897$$

Estimated value s_Δ for σ_Δ:

$s_y = s_\Delta = \sqrt{\ln(V_\delta^2 + 1)} \approx V_\delta = 0,09$ [*input:* (If V_δ is known from prior knowledge)].

Estimated value s_Δ for σ_Δ [(If V_δ is unknown from prior knowledge)]:

$$s_y = s_\Delta = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (\Delta_i - m_y)^2} =$$

$$= \sqrt{\frac{1}{29}[(2,960 - 2,897)^2 + (2,986 - 2,897)^2 + \ldots + (2,602 - 2,897)^2]} = 0,139.$$

Values of k_n for the 5% characteristic value for n = 30 (see tab. D1-EN1990):

$$k_n = \begin{cases} 1,67 & V_x \text{ known} \\ 1,73 & V_x \text{ unknown} \end{cases}$$

Design value of the property X:

$$X_d = \frac{\eta_d}{\gamma_m}\exp[m_y - k_n s_y] = \frac{\eta_d}{\gamma_m}\cdot\exp\left[2,897 - \begin{bmatrix}1,67\\1,73\end{bmatrix}\begin{bmatrix}0,09\\0,139\end{bmatrix}\right] = \frac{\eta_d}{\gamma_m}\cdot\begin{cases}15,6 & V_x \text{ known}\\14,2 & V_x \text{ unknown}\end{cases}$$

having considered already known the ratio η_d/γ_m.

example-end

EXAMPLE 1-I- D8.2 Standard evaluation procedure (Method (a)) - **test 8**

Given: Develop a design model leading to the derivation of a resistance function (see section D8.2-EN1990 - *Standard evaluation procedure (Method (a))*). Check the validity of this model by means of a statistical interpretation of all available test data (see table below)(1). Verify that a sufficient correlation is achieved between the theoretical values and the test data.

[*Reference sheet:* Annex D]-[*Cell-Range: A173:O173-A384:O384*].

r_{ti}	r_{ei}	r_{ti}	r_{ei}
10,5	10,9	18,9	18,4
12,6	12,3	19,4	18,9
14,7	14,9	19,7	19,5
14,9	14,2	20,4	20,8
15,1	14,8	20,8	21,0
15,3	14,7	21,4	21,7
15,8	15,2	21,9	22,0
16,1	15,6	22,5	22,8
16,5	15,5	22,9	23,2
16,9	15,0	23,6	23,9
17,2	16,5	23,9	24,1
17,4	16,9	24,7	25,0
17,8	17,5	25,2	25,5
18,1	18,5	25,9	26,2
18,5	18,3	26,4	25,0

Table 1.3 Sample results (n = 30). Reference Sheet: Annex D. Cell-Range A194:B208 - D194:E208.

(1) r_{ti} theoretical values; r_{ei} experimental values from the tests.

Solution: For the standard evaluation procedure the following assumptions are made:
– the resistance function is a function of a number of independent variables X
– a sufficient number of test results is available
– all relevant geometrical and material properties are measured
– there is no statistical correlation between the variables in the resistance function
– all variables follow either a Normal or a log-normal distribution.

Step 1. Develop a design model, say in general:

$$r_{ti} = A_i \cdot B_i \cdot C_I \cdot D_I \cdot H_I \cdot L_I \cdot M_I \cdot N_I \cdot Q_I \cdot T_I .$$

Step 2. Compare experimental and theoretical values.
The points representing pairs of corresponding values (r_{ti}; r_{ei}) are plotted on a diagram (see data on table 1.3):

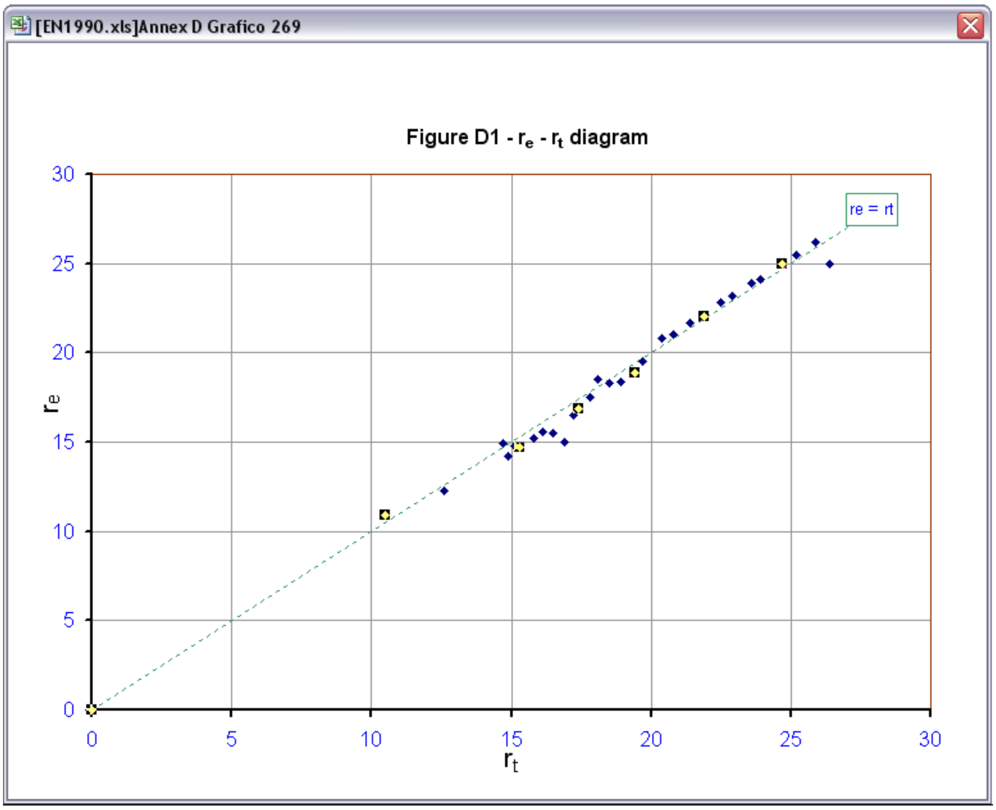

Figure 1.1 Windows screen image: figure D1-EN1990 (r_e - r_t diagram).

As we can see in figure 1.1, all of the points lie on the line $\theta = \pi/4$ (equation $r_e = r_t$). It means that the resistance function is reasonably exact and complete: a sufficient correlation is achieved between the theoretical values and the test data.

Step 3. Estimate the mean value correction factor b.

$$\sum_{i=1}^{n} r_{ei} r_{ti} = 10,5 \times 10,9 + 12,6 \times 12,3 + \ldots + 26,4 \times 25,0 = 11401$$

$$\sum_{i=1}^{n} r_{ti}^2 = (10,5)^2 + (12,6)^2 + (14,7)^2 + \ldots + (25,9)^2 + (26,4)^2 = 11501$$

$$b = \frac{\sum_{i=1}^{n} r_{ei} r_{ti}}{\sum_{i=1}^{n} r_{ti}^2} = \frac{11401}{11501} = 0,991 .$$

Probabilistic model of the resistance $r = br_t\delta$. The mean value of the theoretical resistance function, calculated using the mean values $\underline{X}_m$ of the basic variables, can be obtained from:

$$r_m = br_t(\underline{X}_m)\delta = bg_{rt}(\underline{X}_m)\delta .$$

Step 4. Estimate the coefficient of variation of the errors.

The error term δ_i for each experimental value r_{ei} should be determined from expression (D9-EN1990):

$$\delta_i = \frac{r_{ei}}{br_{ti}} .$$

From which, using the given numerical data into table 1.3, we find (rounding to three decimal places):

$$\delta_1 = \frac{r_{e1}}{br_{t1}} = \frac{10,9}{0,991 \times 10,5} = 1,047 ; \Delta_1 = \ln\delta_1 = \ln(1,047) = 0,046 ;$$

$$\delta_2 = \frac{r_{e2}}{br_{t2}} = \frac{12,3}{0,991 \times 12,6} = 0,985 ; \Delta_2 = \ln\delta_2 = \ln(0,985) = -0,015 ;$$

$$\delta_3 = \frac{r_{e3}}{br_{t3}} = \frac{14,9}{0,991 \times 14,7} = 1,023 ; \Delta_3 = \ln\delta_3 = \ln(1,023) = 0,023 ;$$

...

$$\delta_{30} = \frac{r_{e30}}{br_{t30}} = \frac{25,0}{0,991 \times 26,4} = 0,956 ; \Delta_{30} = \ln\delta_{30} = \ln(0,956) = -0,045 .$$

Substituting the above numerical data into expressions (D.11), (D.12), (D13), we find:

$$\bar{\Delta} = \frac{1}{n}\sum_{i=1}^{n}\Delta_i = \frac{1}{n}\sum_{i=1}^{n}\ln(\delta_i) = \frac{(0,046 - 0,015 + 0,023 + \ldots - 0,045)}{30} = -0,005$$

$$s_\Delta^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - \bar{\Delta})^2 = \frac{(0,046 + 0,005)^2 + (-0,015 + 0,005)^2 + \ldots + (-0,045 + 0,005)^2}{29} = 0,001$$

Coefficient of variation V_δ of the δ_i error terms:

$$V_\delta = \sqrt{\exp(s_\Delta^2) - 1} = \sqrt{\exp(0,001^2) - 1} = 0,032 .$$

Step 5. Analyse compatibility.

The compatibility of the test population with the assumptions made in the resistance function should be analysed. If the scatter of the $(r_{ei}; r_{ti})$ values is too high to give economical design resistance functions, this scatter may be reduced. To determine which parameters have most influence on the scatter, the test results may be split into subsets with respect to these parameters. When determining the fractile factors k_n (see step 7), the k_n value for the sub-sets may be determined on the basis of the total number of the tests in the original series.

Step 6. Determine the coefficients of variation V_{Xi} of the basic variables.

Consider, for example, the design model for the theoretical resistance r_{ti} as represented by the following relation (bearing resistance for bolts):

$$r_{ti} = 2,5 d_i t_i f_{ui} \equiv 2,5\ B_i \cdot C_i \cdot D_i ; (A_i = A = 2,5 = \text{cost}).$$

The resistance function above covers all relevant basic variables $\underline{X}$ that affect the resistance at the relevant limit state. The coefficients of variation V_{Xi} will normally need to be determined on the basis of some prior knowledge. Therefore, let us say:

1) coefficient of variation $V_d = 0,04$ of the basic variable of the bolt's diameter;

2) coefficient of variation $V_t = 0,05$ of the b. v. of the thickness of the connected part;

3) coefficient of variation $V_{fu} = 0,07$ of the b. v. of the ultimate tensile strength of the materials.

Step 7. Determine the characteristic value r_t of the resistance.

The resistance function for j (= 4) basic variables is a product function of the form:

$$r = b r_t \delta = b\{A \cdot B \cdot C \cdot D\}\delta .$$

Coefficient of variation V_r:

$$V_r^2 = (V_\delta^2 + 1)\left[\prod_{i=1}^{j}(V_{Xi}^2 + 1)\right] - 1 = (V_\delta^2 + 1) \cdot (V_a^2 + 1) \cdot (V_d^2 + 1) \cdot (V_t^2 + 1) \cdot (V_{fu}^2 + 1) - 1$$

having considered $V_A = 0$ for the constant $A = 2,5$. Therefore, rounded to two decimal places, we find:

$$V_r^2 = (0,032^2 + 1) \cdot (0^2 + 1) \cdot (0,04^2 + 1) \cdot (0,05^2 + 1) \cdot (0,07^2 + 1) - 1 = 0,01$$

$$V_\delta = \sqrt{\exp(s_\Delta^2) - 1} = 0,032$$

$$V_{rt}^2 = \sum_{i=1}^{n} V_{xi}^2 = V_a^2 + V_d^2 + V_t^2 + V_{fu}^2 = 0 + 0,04^2 + 0,05^2 + 0,07^2 = 0,009 .$$

The number of test is limited (n = 30 < 100). In this case the characteristic resistance r_k should be obtained from [see equation (D.17)-EN1990]:

$$r_k = b g_{rt}(\underline{X}_m)\exp(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\delta Q_\delta - 0,5Q^2) \text{ with:}$$

$Q = \sqrt{\ln(V_r^2 + 1)} = \sqrt{\ln(0,01 + 1)} = 0,100$

$Q_{rt} = \sqrt{\ln(V_{rt}^2 + 1)} = \sqrt{\ln(0,009 + 1)} = 0,095\,;\ \alpha_{rt} = Q_{rt}/Q = (0,095)/(0,100) = 0,95$

$Q_\delta = \sqrt{\ln(V_\delta^2 + 1)} = \sqrt{\ln(0,032^2 + 1)} = 0,032\,;\ \alpha_\delta = Q_\delta/Q = (0,032)/(0,100) = 0,32$.

Values of k_n for the 5% characteristic value for n = 30 (see tab. D1-EN1990):

$$\begin{bmatrix} k_\infty \\ k_n \end{bmatrix} = \begin{cases} 1,64 & \text{for } n \to \infty \\ 1,73 & V_x \text{ unknown} \end{cases}$$

Substituting the numerical data into expressions above, we find the characteristic value of the resistance:

$r_k = r_m \exp(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\delta Q_\delta - 0,5Q^2) =$

$= r_m \exp[-(1,64 \cdot 0,95 \cdot 0,095) - (1,73 \cdot 0,32 \cdot 0,032) - 0,5 \cdot 0,100^2] = r_m \cdot \exp(-0,171) =$

$= r_m \cdot \exp(-0,171) = r_m \cdot 0,84$

Here the characteristic value r_k is represented as being proportional to its mean r_m .

example-end

EXAMPLE 1-J-D8.3 Standard evaluation procedure (Method (b)) - **test 9**

Given: Considering the same assumptions in the example above, determine the design value of the resistance by taking account of the deviations of all the variables.

[*Reference sheet:* Annex D]-[*Cell-Range: A387:O387-A413:O413*].

Solution: In this case the procedure is the same as in D8.2, excepted that step 7 is adapted by replacing the characteristic fractile factor k_n by the design fractile factor $k_{d,n}$ equal to the product $\alpha_R \beta$ assessed at $0,8 \times 3,8 = 3,04$ as commonly accepted (see Annex C-EN1990) to obtain the design value r_d of the resistance.

For the case of a limited number of tests (herein n = 30 < 100) the design value r_d should be obtained from:

$r_d = b g_{rt}(\underline{X}_m) \exp(-k_{d,\infty} \alpha_{rt} Q_{rt} - k_{d,n} \alpha_\delta Q_\delta - 0,5Q^2)$

where:

$k_{d,n}$ is the design fractile factor from table D2 for the case "V_X unknown"

$k_{d,\infty}$ is the value of $k_{d,n}$ for $n \to \infty$ $[k_{d,\infty} = 3,04]$.

The value of $k_{d,n}$ for the ULS design value (leading) is 3,44 (see table D2-EN1990).

Therefore, we get:

$r_d = r_m \exp(-k_{d,\infty} \alpha_{rt} Q_{rt} - k_{d,n} \alpha_\delta Q_\delta - 0,5Q^2) =$

$= r_m \exp[-(3,04 \cdot 0,95 \cdot 0,095) - (3,44 \cdot 0,32 \cdot 0,032) - 0,5 \cdot 0,100^2] = r_m \cdot \exp(-0,315) =$

$= r_m \cdot \exp(-0,315) = r_m \cdot 0,73$

having represented r_d as being proportional to its mean.

Dividing the characteristic value by the design value we obtain:

$$\gamma_R = \frac{r_k}{r_d} = \frac{r_m \cdot 0,84}{r_m \cdot 0,73} \approx 1,15$$

having estimated V_δ from the test sample under consideration (see data in tab. 1.3).

example-end

EXAMPLE 1-K-D8.4 Use additional prior knowledge - **test 10**

Given: Determine the characteristic value r_k of resistance when:

– only one further test is carried out.

– two or three further tests are carried out.

Suppose that the maximum coefficient of variation observed in previous tests is equal to $V_r = 0,09$.

[*Reference sheet:* Annex D]-[*Cell-Range: A419:O419-A438:O438*].

Solution: If only one further test is carried out, the characteristic value r_k may be determined from the result r_e of this test by applying (D.24-EN1990):

$$r_k = r_e \cdot \eta_k = r_e \cdot 0,9\exp(-2,31V_r - 0,5V_r^2) = r_e \cdot 0,9\exp(-2,31 \cdot 0,09 - 0,5 \cdot 0,09^2) = r_e \cdot 0,73$$

where η_k is a reduction factor applicable in the case of prior knowledge.

If two or three further tests are carried out, the characteristic value r_k may be determined from the mean value r_{em} of the test results by applying (D.26-EN1990):

$$r_k = r_e \cdot \eta_k = r_e \cdot \exp(-2,0V_r - 0,5V_r^2) = r_e \cdot \exp(-2,0 \cdot 0,09 - 0,5 \cdot 0,09^2) = r_e \cdot 0,83$$

provided that each extreme (maximum or minimum) value r_{ee} satisfies the condition: $|r_{ee} - r_{em}| \le 0,10 r_{em}$.

example-end

1.5 References [Section 1]

BS EN 1990 - *Eurocode 0: Basis of structural design*, 1 July 2002

European Committee for Standardization (2001) *Eurocode: Basis of Structural Design*, CEN, Brussels, EN 1990

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